Studia Geotechnica et Mechanica, Vol. XXIX, No. 3–4, 2007

VALIDATION OF AN ELASTOPLASTIC THEORY OF THE PRESSUREMETER TEST IN GRANULAR SOIL

J. MONNET

Joseph Fourier University, L3S-R, Domaine Univeristaire,
BP n°53, 38041, Grenoble, Cedex 9, France. E-mail: jmonnet@ujf-grenoble.fr

Abstract: The recent elastoplastic pressuremeter theory of MONNET and KHLIF [12] for the granular soil has been used in several civil engineering constructions. The finite element method allows nowadays the validity of this theory to be checked. In the first part of the paper, we briefly present the interpretation of the pressuremeter test in granular soil and a theoretical expression of the limit pressure. In the second part, we present the analysis of the theoretical limit pressure versus the result of the Mohr–Coulomb non-standard model used by the Plaxis finite element program. As the theory shows that the limit pressure depends on four parameters, we apply a variation of one of these parameters, the others remaining unchanged, and we study the resulting variation of the limit pressure. The theoretical evolution of the limit pressure as a function of each parameter shows a fine agreement with the numerical results.

Streszczenie: Opracowaną przez MONNETA i KHLIFA [12] sprężystoplastyczną teorię testu presjometrycznego dla ośrodków sypkich wykorzystuje się w zagadnieniach inżynierii budowlanej. Potwierdzeniem teorii są rozwiązania, które umożliwia metoda elementów skończonych. W pierwszej części pracy przedstawiono zastosowanie presjometru dla ośrodków sypkich oraz teoretyczny wzór naprężenia granicznego. W drugiej części analizowano teoretyczne naprężenia graniczne, porównując je z wynikami niestandardowego modelu Mohra–Coulomba za pomocą programu metody elementów skończonych Plaxis. Jak wykazuje teoria, naprężenie graniczne zależy od czterech parametrów. Zastosowano zmianę jednego z parametrów i, zakładając niezmienność pozostałych, analizowano zmiany naprężeń granicznych. Otrzymane teoretyczne zmiany naprężeń granicznych jako funkcje każdego z parametrów wykazały dobrą zgodność z rozwiązaniem numerycznym.

Резюме: Разработанная Монэ и Клифом [12] упруго-пластическая теория прессиометрического теста для сыпучих сред используется в вопросах строительного инженерного дела. Подтверждением теории являются решения, которые предоставляет метод конечных элементов. В первой части работы представлено применение прессиометра для сыпучих сред, а также теоретическое уравнение предельного напряжения. Во второй части проведен анализ теоретических предельных напряжений, сравнивая их с результатами стандартной модели Кулона–Мора с помощью программы метода конченных элементов Плаксис. Как показывает теория, предельное напряжение зависит от четырех параметров. Применена замена одного из параметров и, предполагая неизменчивость остальных, анализировли изменение предельных напряжений. Полученные теоретические изменения предельных напряжений как функции каждого из параметров показали хорошую согласованность с численным решением.

1. INTRODUCTION

The pressuremeter is a well-known apparatus (MÉNARD [11]), widely used nowadays for foundation engineering (GAMBIN [9], AMAR et al. [1], CLARKE [2]). Its use,

however, often is based on a set of empirical rules (DTU 13.12.1988 [4], French Standard NF P 94-110 2000 [8], French Standard P 94-250-1 [7]).

A pressuremeter test may be considered as an in situ shearing test because the instrument measures both deformability and shear resistance of soil and the test is performed in situ, in any soil, without sampling. This allows us to avoid the problems of grain size distribution, change in consolidation or remoulding, often encountered in the samples used for laboratory testing.

2. HYPOTHESIS

Following Baguelin et al. (1978), we assume a drained test with an elastic behaviour at low level of shear with two elastic parameters, the Young modulus E and the Poisson ratio ν and a non-standard elastoplasticity with a dilantancy angle (eq. (1)) which is:

$$\Psi = \Phi' - \Phi_{\mu}\,, \tag{1}$$

$$0.8\,\Psi = \Phi' - \Phi_{cv}. \tag{2}$$

The relation between dilatancy and friction angle was also investigated by Bolton (1986), who proposed relation (2), close to the previous one, as the angle Φ_{cv} is larger than the interparticle angle Φ_{μ} (Rowe, 1962; Rowe 1969; Frydman et al., 1973). The use of fixed angles of dilatation and friction is a simplification and it would be preferable to consider these angles as the function of density and pressure. This would, however, result in even more complex mathematics and it is not compatible with the aim of finding simple mechanical characteristics.

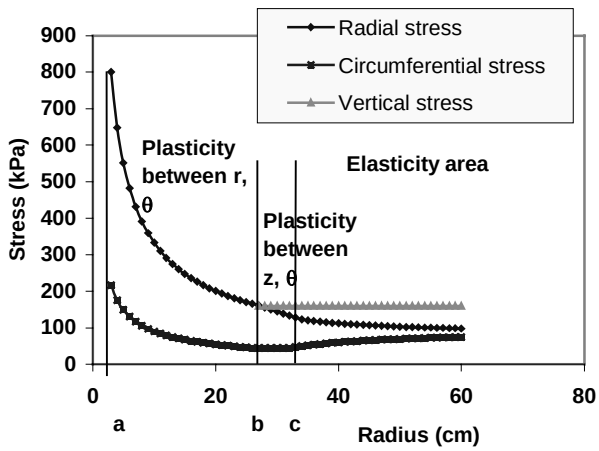

Fig. 1. The three areas around the pressuremeter probe

The Mohr–Coulomb relation gives the failure of the soil:

$$F(\sigma) = (\sigma_1' - \sigma_3') - \sin\Phi'(\sigma_1' + \sigma_3') \,. \tag{3}$$

The non-associated flow rule is:

$$d\varepsilon^p = \xi dH(\sigma')/d\sigma' \,, \tag{4}$$

with the unknown scalar ξ and the plastic potential:

$$H(\sigma') = (\sigma_1' - \sigma_3') - \sin\Psi(\sigma_1' + \sigma_3') \,. \tag{5}$$

Three different areas of soil are considered from the borehole wall to the infinite radius (figure 1).

Plasticity appears between the radial stress σ_r' and the circumferential stress σ_θ' in the horizontal plane (figure 1). This first plastic area extends between the radius *a* (borehole wall) and the radius *b* (external radius of the first plastic area). As shown by WOOD and WROTH [13], plasticity may also appear in the vertical plane between the vertical stress σ_z' and the circumferential stress σ_θ' (figure 2) in an area between the radii *b* and *c* (external radius of both plastic areas).

An elastic area extends beyond the radius *c*.

3. THEORETICAL ELASTOPLASTIC EQUILIBRIUM AROUND PRESSUREMETER

3.1. GLOBAL EQUILIBRIUM WITH ONLY ONE PLASTIC AREA

The global elastoplastic equilibrium was found with one plastic area (MONNET and KHLIF [12]). The continuity of stress between the different areas allows evaluating two internal constants of the model, C_1 and δ. A general condition of the equilibrium between stress and strain, which is the general form of the pressuremeter equation for one plastic area, is then:

$$\ln\left[\frac{u_a}{a}.(1+n) - C_1\right] = \delta.\ln(p) - \delta.\ln\left[\frac{2.K_0.\gamma.z}{(1+N)}\right] + \ln\left[K_0.\gamma.z.\frac{(1-N).(1+n)}{2.G.(1+N)} - C_1\right], \tag{6}$$

with $$\delta = \frac{1+n}{1-N} \quad \text{and} \quad C_1 = \frac{K_0.\gamma.z + (1-N).(n-1)}{2.G.(1+N)}, \tag{7}$$

with $N = (1-\sin\Phi')/(1+\sin\Phi')$ and $n = (1-\sin\Psi)/(1+\sin\Psi)$. (8)

We can obtain the limit pressure p_l for a volume of the probe which is double the initial one. The borehole strain at the borehole is then equal to $\sqrt{2}-1$, and the conventional limit pressure is then:

$$p_l' = \frac{2.K_0.\gamma.z}{(1+N)}.\sqrt[\delta]{\frac{[(1+n).(\sqrt{2}-1)-C_1].2.G.(1+N)}{K_0.\gamma'.z.[(1-N).(1+n)-2.G.C_1.(1+N)]}}. \quad (9)$$

This relation is quite different from that of AMAR's et al. [1], which is based on the Ménard experimental correlations:

$$p_l' = 250.\left[2^{\left(\frac{\phi-24}{4}\right)}\right] + K_0.\gamma.'z. \quad (10)$$

The Ménard relation was derived from many pressuremeter tests. Theoretical considerations show that the main shearing takes place between the radial stress σ_r' and the circumferential stress σ_θ', which lie in the horizontal plane. For a granular soil, the plasticity condition shows that the level of shearing is proportional to the level of stress applied to the shearing surface. For the pressuremeter test, the vertical stress is normal to the horizontal surface, so σ_z' can be considered to be the stress along which shearing takes place. As the limit pressure is linked with a particular value of the shearing stress, it must also be proportional to the vertical stress, which is obtained from equation (9). The Ménard equation (10) seems to fit these considerations only for a mean depth close to 12 m. For a test carried out closely to the surface, it seems to underestimate the friction angle, while for test conducted very deeply it seems to overestimate the friction angle. Futhermore, the Ménard relation does not take into account the nature of the soil and the variation of the interparticle angle of friction. It overestimates friction angle for loams, which have smaller interparticle angle of friction than sands and gravels.

3.2. GLOBAL EQUILIBRIUM WITH TWO PLASTIC AREAS

The continuity of stress between the three different areas allows the evaluation of two internal constants C_1 and δ of the model. We obtain a general condition of the equilibrium between stress and strain, which is the general form of the pressuremeter equation with two plastic areas:

$$\ln\left[\frac{u_a}{a}.(1+n)-C_1\right] = \delta.\ln(p) - \delta.\ln(\gamma.z) + \ln\left[(1-K_0).\gamma.z.\frac{(1+n)}{2.G} - C_1\right], \quad (11)$$

with

$$C_1 = \frac{n.\left(\frac{u_a}{a}\right).(1+n).\left(\frac{\gamma.z}{p}\right)^{\delta} + (1+n).(N-K_0).\frac{\gamma.z}{2.G}}{1+n.\left(\frac{\gamma.z}{p}\right)^{\delta}}. \tag{12}$$

Table

Mechanical parameters used in the numerical analysis

Parameter studied	E (MPa)	ν	G (MPa)	K_0	σ_z (kPa)	Φ_μ (degree)	Φ' (degree)
σ_z – 1 zone	40	0.42	14.12	0.724	50–1000	27.8	30
σ_z – 2 zones	40	0.30	7.26	0.429	50–1000	27.8	30
G – 1 zone	10–100	0.42	3.52–35.21	0.724	250	27.8	30
G – 2 zones	10–100	0.30	3.84–38.46	0.429	250	27.8	30
K_0 – 1 zone	40	0.37–0.5	14.6–13.33	0.587–1	250	27.8	45
K_0 – 2 zones	40	0.2–0.36	16.7–14.71	0.25–0.56	250	27.8	45
Φ_μ – 1 zone	40	0.42	14.09	0.724	250	10–30	30
Φ_μ – 2 zones	40	0.30	15.38	0.429	250	10–30	30
Φ' – 1 zone	40	0.42	14.29	0.724	250	27.8	30–45
Φ – 2 zones	40	0.30	15.38	0.429	250	27.8	30–45

The value of the coefficient C_1 is usually close to one hundredth of the borehole strain. For example, with the mean values of the numerical analysis (the table) and at an applied pressure of 500 kPa, with a borehole deformation of $4.27 \cdot 10^{-2}$, the value of C_1 is $1.45 \cdot 10^{-3}$, which is 3.4% of the radial deformation. This is very small and can be neglected, and leads to a linear relation between the logarithm of the borehole strain at the borehole wall and the pressure applied by the pressuremeter. The proportionality between these two variables was first shown by HUGHES et al. [10] with the use of Rowe's dilatancy theory. Such a relation allows the determination of the slope δ of the straight line between the variables, which is a function of Φ', the angle of internal friction, and Φ_μ, the interparticle angle of friction. If Φ_μ and δ are known, Φ', the angle of internal friction of the soil, can be uniquely and accurately determined.

We can obtain the limit pressure p_l for a volume of the probe which is double the initial one. The borehole strain at the borehole is then equal to $\sqrt{2}-1$ and the conventional limit pressure is then:

$$p_l' = \gamma.z.\sqrt[\delta]{\frac{[(1+n).(\sqrt{2}-1)-C_1].2.G}{[(1-K_0).(1+n).\gamma'.z-2.G.C_1]}}. \tag{13}$$

The difference between the two cases is linked with the value of the radial stress at the radius c of the external area of plasticity. For a single plasticity between r and θ, the radial stress must be higher than the vertical stress, i.e., $\sigma'_{rc} > \sigma'_z$ and this leads to the WOOD and WROTH [13] relation between K_0 and Φ':

$$K_0 \geq \frac{1}{(1+\sin\Phi')} . \tag{14}$$

4. NUMERICAL ANALYSIS FOR THE ELASTOPLASTIC THEORY

4.1. METHOD USED TO DETERMINE THE CHARACTERISTICS OF THE SOIL

The measurement of the slope (figure 2) of the curve representing a linear relationship between the logarithms of pressure and borehole strain at the borehole wall allows the determination of the angle of internal friction using equations (1), (6)–(8). This value is then inserted into equations (6), (11) to plot a theoretical pressuremeter

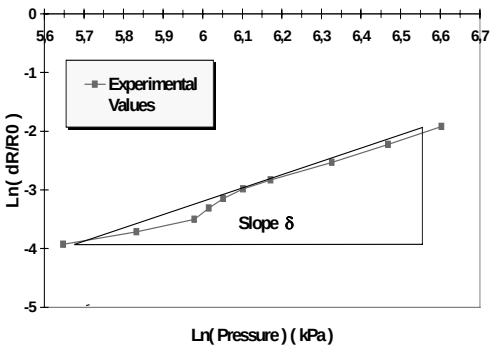

Fig. 2. Linear transformation of the pressuremeter relationship for a test in a gravel site

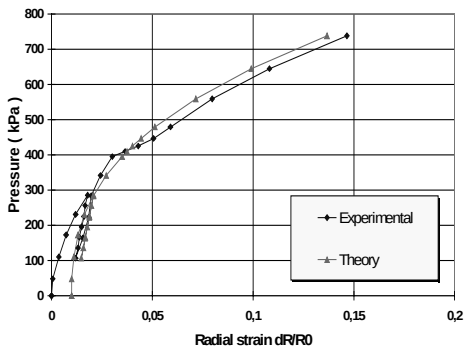

Fig. 3. Control of the stress–strain parameters for a test in a gravel site

curve, which is compared with the experimental one (figure 3). The fit between experiment and theory must be exact in the unloading–reloading sequence so that the Young modulus is properly controlled. This condition introduces an initial translation (figure 3) of the theoretical curve, which can be considered as the plastic effect induced by the pre-drilled technique. A correct fitting of the two curves must be exact above the creep pressure to control the angle of friction. The greater the angle of friction, the higher the theoretical curve within the representation of figure 3. An accurate matching of the curves means that the set of stress–strain parameters is correct for the theoretical representation of the pressuremeter test.

The correct adjustment in the theoretical and numerical curves from the beginning to the end involves many points, and the objective to carry out a variation of each parameter involves many more points. It is not possible to manage such a large number of results, so it was decided to simplify this study by using only one point of the pressuremeter curve, which is a conventional pressuremeter limit. This value can be obtained by the direct theoretical and numerical analyses for an expansion of the pressuremeter probe, which is double of the initial volume.

The common set of parameters used is as follows: E = 40 MPa, ν = 0.42, σ_z = 250 kPa, Φ_μ = 27.8, Φ' = 30°. We then apply a perturbation to one of these parameters, for example, to the Young modulus so that the shear modulus varies, or to the Poisson ratio so that K_0 varies, or to the vertical stress, or to the friction angle, or to the interparticle angle of friction, while keeping the others parameters constant (the table). For the study of K_0, it is necessary to change also the angle of friction so that two plastic areas appear. In the figures, CTRE4 is a result of equation (10), which can be found in the report of European Regional Technical Committee 4 by AMAR et al. [1].

4.2. INFLUENCE OF THE VERTICAL STRESS

The theory takes into account the vertical stress as the intermediate stress between the radial stress and the circumferential one. It shows that shearing takes place under the conditions of stress whose value is close to that of the vertical stress. For granular

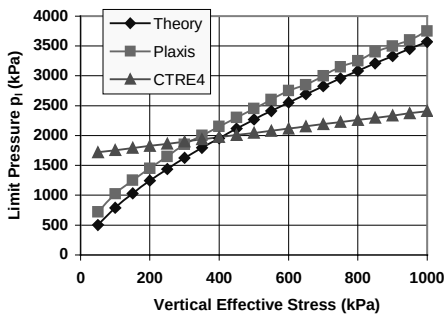

Fig. 4. Influence of the vertical stress on the limit pressure in a test with one plastic zone

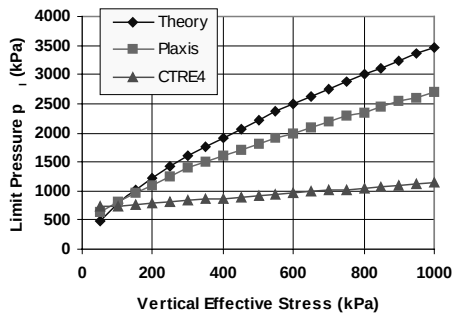

Fig. 5. Influence of the vertical stress on the limit pressure in a test with two plastic zones

soil, the limit pressure is then a function of the vertical stress and when the vertical stress increases, the friction between the radial stress and the circumferential stress leads to an increase of the limit pressure. This variation is described by theoretical equations (9), (13), where the vertical stress is a multiplicative factor in the theoretical

limit pressure. The finite element method (figures 4, 5) shows the same variation of the limit pressure. The difference between the theory and the Plaxis calculus is close to 250 kPa or 100 kPa and stays constant during a variation of the vertical stress from 0 to 500 kPa, which allows us to validate the influence of the vertical stress on the limit pressure.

The Ménard correlation equation (10) assumes that the net limit pressure (the difference between the limit pressure and the horizontal at rest pressure) does not depend on the vertical stress and appears far from the numerical results (figures 4, 5, curve CTRE4). The estimation of the limit pressure by this relation should be used only in the range of vertical stress between 100 and 200 kPa, where the differences in numerical results remain in a narrow range.

4.3. INFLUENCE OF THE SHEARING MODULUS

The limit pressure is the value of the pressure linked with the volume of the probe, which is twice the initial one. If the soil is stiffer, at a defined value of the pressure, the deformation of the soil should be smaller, and the deformation to twice the initial volume should be reached at a high value of the pressure. On the other hand, for a soft

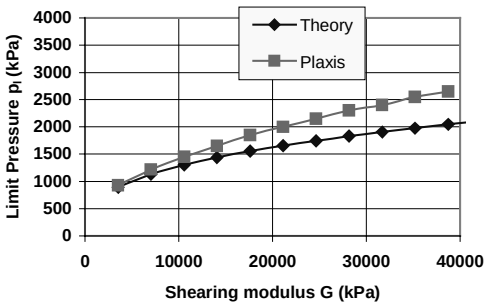

Fig. 6. Influence of the shearing modulus on the limit pressure in a test with one plastic zone

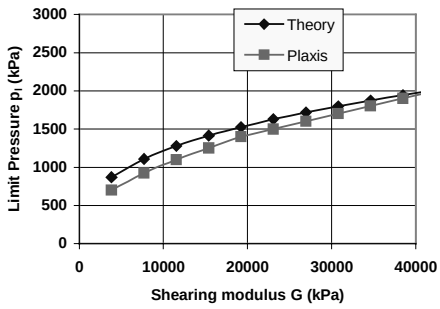

Fig. 7. Influence of the shearing modulus on the limit pressure in a test with two plastic zones

soil, at a defined value of the pressure, the deformation of the soil should be larger, and the deformation to twice the initial volume should be reached at a low value of the pressure. This evolution is predicted by the theory, and we can see (figures 6, 7) that the shearing modulus has an increasing influence on the limit pressure. As the shearing modulus increases, the limit pressure increases. Furthermore, the theory can predict with the accuracy of 20% the limit pressure found by the Plaxis program.

The correlation relation (10) of Ménard assumes that there is no influence of the shearing modulus on the limit pressure and it is not plotted.

4.4. INFLUENCE OF THE FRICTION ANGLE

The angle of friction acts as a resistance factor of the deformation of the soil, and when the friction angle increases the limit pressure also increases. This is predicted by the theory, where the limit pressure is a function (through the variable N) of the friction angle. This variation is reproduced by the finite element analysis made by Plaxis, and an exact fitting between the theory and the numerical result is obtained with an error smaller than 12% of the value of the limit pressure (figures 8, 9), which validates the theory for the variation of the friction angle.

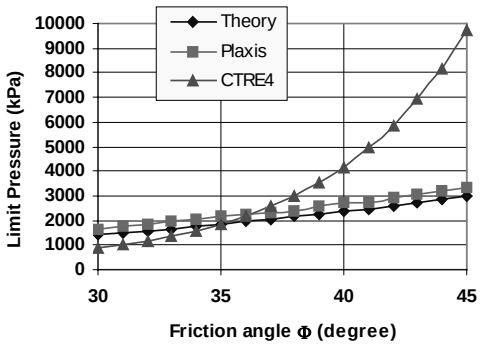

Fig. 8. Influence of the friction angle on the limit pressure in a test with one plastic zone

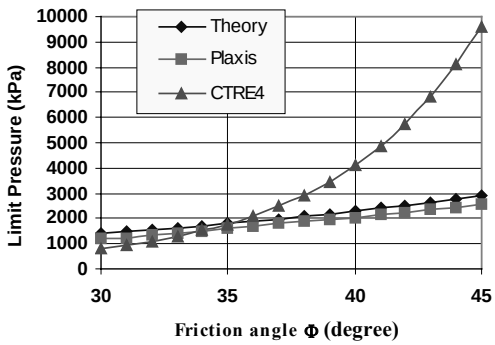

Fig. 9. Influence of the friction angle on the limit pressure in a test with two plastic zones

If we consider the correlation relation of Ménard (10), it can be seen that this gives an overestimation of the angle of friction below 35° and a poor estimation of the friction angle with a large underestimation of above 35° for the two cases analysed (figures 8, 9, curves CTRE4).

4.5. INFLUENCE OF THE INTERPARTICLE ANGLE OF FRICTION

The interparticle angle of friction varies from 10° for clay to 30° for sand and gravel. The theory shows that it influences the dilatancy equation (1). It can be assumed that an increase of the dilatancy increases the limit pressure, because the expansion of the soil increases the volume of the plastic area around the probe. This theoretical phenomenon is described by the theory (figures 10, 11), and when the dilatancy is large (equal to 20° for an interparticle angle of 10° related to a friction angle of 30°), the limit pressure is also high, while when the dilatancy is small (equal to 0° for an interparticle angle of 30° related to a friction angle of 30°) the limit pressure is also low. The difference between the theory and the finite element results stays in a small range and validates the influence of the interparticle angle of friction on the limit pressure.

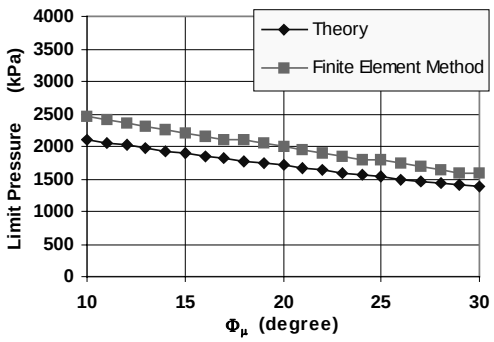

Fig. 10. Influence of the interparticle angle on the limit pressure in a test with one plastic zone

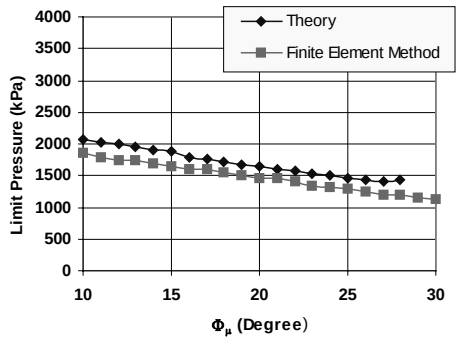

Fig. 11. Influence of the interparticle angle on the limit pressure in a test with two plastic zones

There is no influence of the interparticle angle of friction on the limit pressure in Ménard's equation, which is not drawn here.

4.6. INFLUENCE OF AT REST PRESSURE COEFFICIENT K_0

The at rest pressure coefficient K_0 has a slight influence on the limit pressure. Its small increase should increase the horizontal at rest pressure coefficient and consequently the limit pressure. This evolution is predicted by the theory for one plastic area (figure 12). In the second case studied (figure 13), the theoretical limit pressure shows no variations, when the finite element solution shows a small increase in limit pressure. However, the difference compared with the finite element calculation is in a narrow range for a value of K_0 higher than 0.3, which is related to the more common

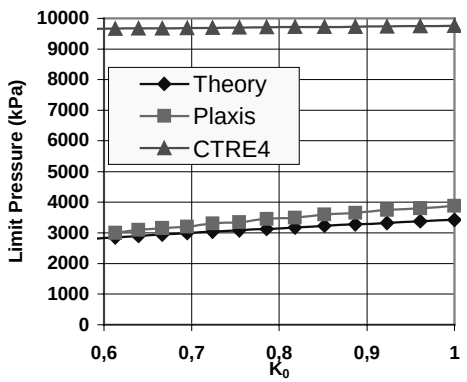

Fig. 12. Influence of the coefficient K_0 on the limit pressure in a test with one plastic zone

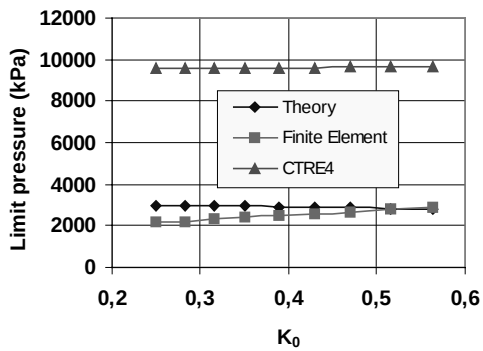

Fig. 13. Influence of the coefficient K_0 on the limit pressure in a test with two plastic zones

case where Poisson's value is higher than 0.23. It can be seen that the relation proposed by Ménard differs significantly from the theory. This is explained by the values used for the calculation, with an angle of friction of 45°, which is needed to have two different plastic zones. In the previous part (figures 8, 9), we have seen that relation (10) widely underestimates the friction angle for its values larger than 35°.

5. CONCLUSION

We have presented a numerical analysis of the theory developed in order to interpret the results of pressuremeter test, which takes into account the vertical and the horizontal non-standard elastoplastic equilibria around the pressuremeter probe. The plasticity may occur between the radial stress and the circumferential stress and between the vertical stress and the circumferential stress.

The theory shows that five mechanical parameters have an influence on the pressuremeter results in a granular soil (vertical stress, shearing modulus, friction angle, interparticle angle of friction, coefficient of at rest pressure) and that the conventional limit pressure is a function of these parameters. The numerical calculation of the pressuremeter test using the Plaxis code has been made with a variation of one of these variables, the others remaining unchanged. The numerical results show the same variation as the theory for each variable and a close agreement with the Plaxis results. This validates the effect of these parameters on the pressuremeter results and shows the influence of the vertical stress, the shearing modulus, the friction angle and the dilatancy (through the interparticle angle of friction) on the conventional limit pressure. Furthermore it shows that plasticity may appear in the vertical plane between the vertical and circumferential stresses, which decreases the limit pressure.

REFERENCES

[1] AMAR S., CLARKE B.G.F., GAMBIN M., ORR T.L.L., *The application of pressuremeter test results to foundation design in Europe*, European Regional Technical Committee 4, Pressuremeters, 1991, A.A. Balkema, 1–24.

[2] CLARKE B.G., *Pressuremeter testing in ground investigation*. Part I. *Site operations*, Geotechnical Engineering, 1996, 119, 96–108.

[3] CLARKE B.G., *Pressuremeter testing in ground investigation*. Part II. *Interpretation*, Geotechnical Engineering, 1997, 125, 42–51.

[4] DTU 13-12 1988. *Règles pour le calcul des foundations superficielles*, P 11-711, AFNOR.

[5] FAHEY M., *Expansion of a thick cylinder of sand*: *a laboratory simulation of the pressuremeter test*, Geotechnique, 1986, 36, 397–424.

[6] French Standard ISO 9000-3 1994, *Normes pour le management de la qualité et de l'assurance de la qualité* – Partie 3: *Lignes directrices pour l'application de l'ISO 9001 au développement, à la mise à disposition, à l'installation et à la maintenance de logiciel*, AFNOR.

[7] French Standard NF P 94-250-1 1996, *Eurocode 7 Calcul géotechnique*, AFNOR.

[8] French Standard NF P 94-110 2000, *Essai pressiométrique Ménard*, AFNOR.
[9] GAMBIN M., *Vingt ans d'usage du pressiomètre en Europe*, Congrès Européen de Mécanique des Sols et des Travaux de Fondation, Brighton, 1979.
[10] HUGHES J.M.O., WROTH C.P., WINDLE D., *Pressuremeter tests in sand*, Geotechnique, 1977, 27, 4, 455–477.
[11] MÉNARD L., *An apparatus for measuring the strength of soils in place*, Master of Science Thesis, University of Illinois, 1956.
[12] MONNET J., KHLIF J., *Etude théorique et expérimentale de l'équilibre élasto-plastique d'un sol pulvérulent autour du pressiomètre*, Revue Française de Géotechnique, 1994, 67, 3–12.
[13] WOOD D.M., WROTH P.C., *Some laboratory experiments related to the results of pressuremeter tests*, Geotechnique, 1977, 27, 2, 181–201.