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An analytical model to predict water retention curves for granular materials using the grain-size distribution curve

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Abstract: The present work aims to propose a new analytical model intended to predict the water retention curves for granular materials based on data from tensiometric tests. Different analytical models have been used for the evaluation of soil water retention curves so far. It should be noted that the proposed model considers only one criterion in the selection of soils. This criterion is the physical property of particle distribution curve that can be used to determine the values of D_{50} and C_U. In this study, the pore-access size distribution is investigated considering the effect of the coefficient of uniformity of sandy soils that were prepared with different density indexes (0.5, 0.7, and 0.9). Moreover, the proposed model equation is based on the physical properties of soil. This equation made it possible to describe the water retention curve and to estimate the pore-access size distribution without performing any experimental tests. The findings allowed asserting that the uniformity of the particle size curves corresponds to a good uniformity of the pore-access size distribution. In addition, it was revealed that the suction increased as the density index went up, which matches well with the experimental data. Moreover, it may clearly be noted that the distinctive retention properties of unsaturated soils can be observed on the above-mentioned curves. Further, it was found that the ratio of the grain size over the pore-access size increased as the uniformity coefficient augmented.

Keywords: sand; mean grain size; mean pore size; density index; model; pore size distribution; water retention curve.

1 Introduction

Over the past few years, soil suction has been progressively inquired into for the purpose of studying unsaturated soils. Unsaturated soil suction parameter can be of great interest to researchers, which it has a significant impact on the mechanical behavior of soils. Regarding soil conditions, the mechanical behavior of soils is dependent on the movement and balance of water. Consequently, the unsaturated soil suction parameter has to be measured with satisfactory accuracy.

It is worth mentioning that soil suction measurement is widely applied in a high number of domains such as pavement engineering, shallow foundations, natural slopes, rainfall infiltration, as well as in environmentally oriented projects like waste confinement or nuclear storage facilities (Fredlund et al. [23]; Oberg, [48]; Zhan and Ng. [64]; Liang et al. [37]). Previous findings have indicated that features like the hydraulic conductivity, shear strength, compressibility, and swelling potential of unsaturated soils, all depend on suction, which means that the measurement of the suction parameter is of primary importance.

It is widely admitted that soil water content changes, and therefore, appropriate methods are needed to carry out accurate soil *suction* measurements that allow determining the water retention curve. In most geotechnical tests, it is highly recommended to control or impose suction in order to evaluate the hydromechanical behavior of soils. Over the past few years, several studies have been conducted for the purpose of highlighting the importance of matric suction in unsaturated soils (Jamagne et al. [34]; Gupta and Larson. [28]; Rawls et al. [52]; De Jong et al. [15]; Rawls and Brakensiek. [51]; Saxton

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et al. [54]; Vereecken et al. [63]; Bastet et al. [3]; Lilly et al. [38]; Minasny et al. [42]; Bigorre. [5] ; Dexter and Bird. [18]; Wösten et al. [60]; Nemes. [45] and [46]; Pachepsky and Rawls. [49]; Tomasella et al. [55]; Bruand et al. [9]; Morvan et al. [44]).

A number of studies have previously reported that suction variation engenders tremendous modifications in the physical and mechanical properties of soil ((Escario and Saez. [22]; Delage. [50])).

Furthermore, many annotated *bibliographies*, relating to methods and tools for predicting the hydraulic properties of soils like water retention properties and hydraulic conductivity, have been released so far (Bastet et al., [4]; Wösten et al. [60]; Cornelis et al. [13]; Donatelli et al. [20]).

In addition, it is useful to indicate that, over the last few decades, a very large number of studies have investigated the properties of water retention. Unfortunately, it was quite complicated to measure these properties due to the requirements regarding sample storage, and also because the measurement protocols were pretty inconvenient. It should be noted that some early studies were carried out to develop methods to be utilized in assessing some essential geotechnical characteristics of soil like the void ratio, coefficient of curvature, and coefficient of uniformity.

Nowadays, it is widely admitted that the water retention curve is a fundamental hydraulic tool that can be used to define the degree of saturation corresponding to a particular suction in unsaturated soils. Water retention is a property that can affect the mechanical behavior of soil. It is viewed as one of the most important instruments to evaluate the physical quality of soil and to enhance its management as well. The water retention curve represents the relationship between the water content and suction.

Furthermore, the pedotransfer functions are predictive functions of a number of soil characteristics based on data from soil surveys. These functions are indeed viewed as heterogeneous instruments that can be employed in assessing soil properties, and primarily those related to soil water retention.

The term "*Transfer function*" was first introduced, in the form of a mathematical expression, by Bouma and Van Lanen [7]. These authors showed the relationship existing between soil characteristics, i.e., particle size composition, organic carbon content, bulk density, etc., and soil behavior, i.e., water retention, hydraulic conductivity, etc.

As for the expression "*Soil transfer function*," which provides important information on soils, it has been *established quite recently* (Bouma. [8]; van Genuchten and Leij. [61]). According to the literature, numerous studies were carried out in the 1970s and 1980s in relation with this topic. Most of them were aimed at developing possible relations between matric *suction* and the volumetric *water content*. *The findings allowed developing* many interesting models that dealt with soil transfer functions.

Nowadays, no one denies the fact that soil models are fundamental tools that can be highly helpful in performing numerous laboratory experiments in various geotechnical fields, because conducting direct measurements to record complete data sets are quite complicated, time-consuming, and costly.

It should also be noted that *multiple* types of *models* and associated *modeling* for improved prediction of the water retention curve, during the drying process, can be found in the literature. Moreover, over the past years, various models representing the relationship between basic soil characteristics and soil suction have been developed (van Genuchten. [62]; Fredlund and Rahardjo. [24]; Garbulewski and Zakowicz. [26]; Georgiadis et al. [27]; Hoyos and Arduino. [31]; Rojas. [53]). A large number of these models use several parameters that can be assessed using the full particle size distribution curve, which makes even simple geotechnical applications quite complicated.

In order to overcome this problem, an analytical model for the prediction of water retention curves of granular materials, based on tensiometric test data, has been developed and is proposed in the present work. In this model, the water retention curves are predicted for a given soil granulometry for which different water retention curves can be represented as a function of the density of the specimen. Recently, this property is studied and mentioned on the reported models of Chen et al. [11].

The proposed model, which is based on *van Genuchten's water retention model* (1980), was established for the purpose of investigating the water retention parameters of sandy soils, with the introduction of some simplifications and improvements on various points. A semi-statistical approach, using key characteristics of a granular material, was used in this model for the prediction of water retention curves based on the particle size distribution, using solely the uniformity coefficient C_u. The effect of the density index I_D on the water retention curve was also investigated.

It is worth noting that some interesting experimental results, which have previously been found by other researchers, were taken from the literature (Feia et al. [25]).

Furthermore, a parametric study of empirical models was carried out in order to identify the hydraulic properties to be used in this modeling. The first step of

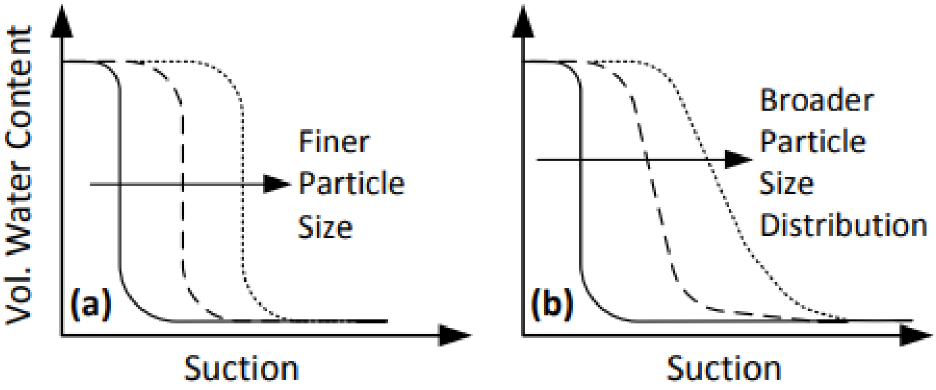


Figure 1: Conceptual diagram showing the effect of **(a)** median particle size of uniform sand and **(b)** width of particle size distribution, on the shape of the soil-water characteristic curve (SWCC) of sand (Craig H. Benson et al. [14]).

this parametric study consisted in estimating the values of the model parameters and fitting the retention curves. Next, these parameters were thoroughly studied, and the sensitivity of the retention curve to the variations in these fitting parameters was profoundly examined.

It is widely admitted that knowing the pore-access size distribution of soils can certainly be of great importance for multiphase flow modeling and pore-scale modeling of transport phenomena. In this context, various applications can be found in groundwater flow hydrology, eco-environmental engineering for soil contamination problems, civil engineering for the design of embankment dam filters, and petroleum engineering for modeling the *produced water re-injection* process in oil reservoirs.

The proposed model was utilized to predict the water retention curves and also to estimate the pore-access size distribution of four different types of sandy soils, without conducting any experiment, using two parameters which are the uniformity coefficient C_u and the density index I_D.

2 Presentation of the proposed model

2.1 The water retention curve in van Genuchten's model

Ji-Peng et al. [35] have defined based on the classical van Genuchten model (van Genuchten [62]), the effective degree of saturation equation by

$$S_e = \left(1 + \left(\frac{s}{\alpha}\right)^n\right)^{\frac{1}{n}-1} \qquad (1)$$

Knowing that the effective degree of saturation is defined as describing the volume of water partially filling the soil microporosity. This effective degree of saturation defines the proportion of the prevailing suction that actually contributes to the effective stress. where *s* is the soil suction, α is a parameter related to the air entry value, and n is a parameter linked to the slope of the water retention curve.

It was revealed that soils with big pores desaturate at smaller suction values and hold back less water for specific matrix suction in comparison with soils with smaller pores, as illustrated in Figure 1a. In this context, Hillel [30] found out that the moisture content of soils with a wide pore size distribution changes progressively as suction changes (Figure 1b). In addition, soils with large pores have a larger parameter n, while those with a wide pore size distribution show a smaller n. Similarly, Tinjum et al. [56] indicated that soils containing minerals, which possess a small surface charge and a small affinity for hydration, possess reduced water content at specific matrix suction.

As is generally known, materials containing small amounts of fines, like clean sand and some other coarse materials, exhibit a close relationship between the particle size distribution, the pore size distribution, and the soil water characteristic curve (Yang et al. [57]; Aubertin et al. [2]; Jaafar and Likos. [33]). In addition, Juang and Holtz [36] found out that as the particle size becomes finer, the pore size becomes smaller. Likewise, it was revealed that, in general, sands with a broader particle distribution possess thinner pores because of the high packing efficiency, which engenders a broader particle size distribution. Based on the above results, and considering Young-Laplace's equation, one may therefore

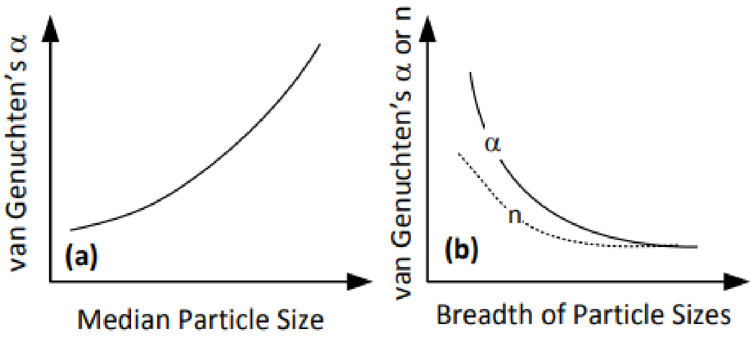


Figure 2: Conceptual diagram presenting the effect of **(a)** the median particle size of uniform sand, and **(b)** the breadth of particle size distribution, on the parameters α and n (Craig H. Benson et al. [12]).

assert that high suction levels are needed to desaturate the pores. Consequently, it can be concluded that, for a given suction of the soil matrix, sands having a broad particle size distribution hold back more water. It is useful to indicate that the pore size diversity augments with the particle size. Moreover, as the distribution becomes wider, the soil-water characteristic curve (*SWCC*) shows a more progressively varied slope.

Consequently, the parameters α and n must change in a consistent manner with the median particle size and the breadth of particle sizes in sand. As is clearly illustrated in Figure 2a, the parameter *α* should decrease as the median particle size diminishes and the width of particle sizes rises, while the parameter n decreases as the breadth of particle sizes goes up, as is shown in figure 2b.

2.2 Formulation of the proposed model

Terzaghi et al. [58] revealed that the three particle sizes d_{10}, d_{30}, and d_{60} relating, respectively, to 10%, 30%, and 60% of passing through the corresponding sieves are the most important particle sizes commonly needed for the *description* and *classification* of soils. Moreover, it is well-known that the shape of the granular distribution can be determined using the coefficient of curvature ($C_c=d^2_{30}/(d_{60}\, d_{10})$), the coefficient of uniformity ($C_u=d_{60}/d_{10}$), in addition to a measure of the mean particle size.

Furthermore, it must be emphasized that dimensional analysis, which is based on the Pi-theorem of Buckingham [10], is extensively utilized in technological analysis and in solving applied science problems for the purpose of identifying the essential factors and making the physical relationships simpler. Regarding the water retention curves, it is important to know that the effective degree of saturation (S_e) depends on the particle size distribution, on suction. In addition, any possible alterations in the void ratio of sandy soils may be considered as quite small. Therefore, the effective degree of saturation (S_e) can be expressed as a function of the dimensionless quantities C_u, C_c, d_{60}, and γ witch γ is a parameter of the model. In addition, Buckingham's Pi theorem can then be applied and S_e can be made simpler and written in the following form as

$$S_e = f(C_u\,, d_{60}\,, I_D\,, s\,, \alpha\,, \Upsilon) \tag{2}$$

Furthermore, according to Ji-Peng Wang et al., [35], the parameter C_c depends directly on C_u. It is worth recalling that C_u is a physical parameter that depends on d_{60}. Consequently, the previous expression may be expressed in a simpler form as

$$S_e \approx f'\left(C_u\,, \frac{sC_u}{\Upsilon}\right) \tag{3}$$

On the other hand, using the normalized suction ($s^*=s.C_u/\gamma$) and the normalized α ($\alpha^*= \alpha.C_u\,/\,\gamma$), van Genuchten's equation may be rewritten as

$$S_e = \left(1 + \left(\frac{s^*}{\alpha^*}\right)^n\right)^{\frac{1}{n}-1} \tag{4}$$

It was shown on the drying path that the degree of saturation remains constant and equals to 1 until the suction reaches the air entry value (AEV). Then, when it reaches the air entry value (AEV), air bubbles enter the water phase. Afterward, the degree of saturation drops considerably as the suction increases further, and the large pores begin to flow while the small ones remain still filled with water, in such a way that the water and air phases become continuous. In addition, it was found that the water phase exists in the form of isolated water bridges and adsorption layers when the suction value gets very

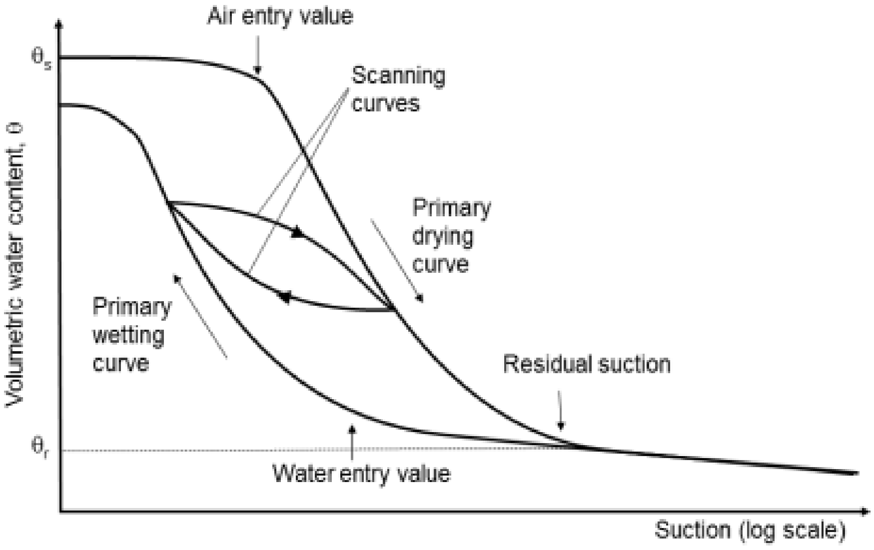


Figure 3: Typical soil water retention curve (Toll [59]).

large. It should be mentioned that the change in suction does not have much effect on the degree of saturation, which corresponds to the residual state. Moreover, the soil-water retention curve (SWRC) may flatten for very small suction values, with the corresponding residual water content. Additionally, any alterations in the volumetric water content would engender an increase in suction. It is ought to be mentioned that inside the residual zone and beyond the residual suction, water is retained as water that was adsorbed on soil particles (McQueen and Miller. [41]) rather than as capillary water maintained by the action of *capillary* forces created by *water menisci*, as is clearly displayed in figure 3.

Based on the above findings, it was deemed interesting to introduce the residual saturation degree in the proposed model in order to develop an expression for the effective degree of saturation S_e that can be used to describe the water retention behavior as

$$S_e = \left(1 + \left(\frac{s^*}{\alpha^*}\right)^n\right)^{\frac{1}{n}-1} - S_{er} \tag{5}$$

where s* is the normalized suction, with $s^* = s.c_u$; α^* is the normalized α, with $\alpha^* = \alpha.c_u$.

Note that α and n are the parameters of the model.

Also, γ is a parameter of the model which is equal to 1, and S_{er} is the degree of residual saturation.

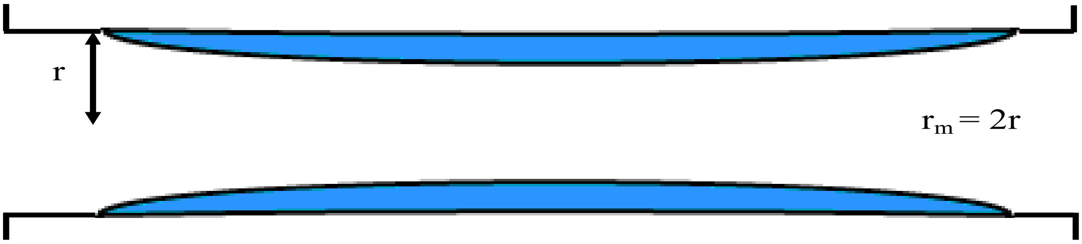
It is worth noting that the model is ought to be fitted to experimental data in order to assess the model parameters which are based on the data obtained from various tests that were carried out using the tensiometric method previously established by Feia et al. [25], and Della and Feia. [47].

3 The tensiometric test

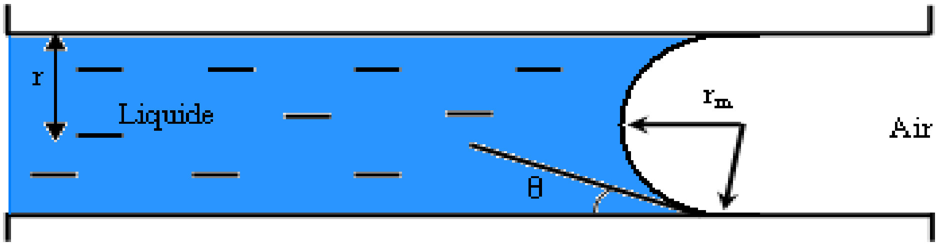
The water retention curve of a sand specimen was estimated using an experimental technique that is similar to the hanging column method (ASTM D6836). This technique is ordinarily used for the assessment of the soil water retention curve (Arya et al. [1]; Likos and Jaafar [39]). It is also interesting to note that it is possible to assess the pore-access size distribution from the water retention curve using the Young-Laplace law which associates suction to the pore-access radius. Note that the main principle behind this method is similar to that utilized in mercury intrusion porosimetry. It was suggested that it is quite possible to apply this method using a simple apparatus with a traditional procedure for the preparation of the specimens.

3.1 Principle of the method

The flow model under consideration adopts a pore network with parallel cylindrical tubes in which the water flows along the direction of the cylinder axis. This assumption corresponds to the drying path when the pores are initially filled with liquid water. Regarding the humidification path, the condensation of the water vapor



a) Wetting process



b) Drying process

Figure 4: Explanatory diagram of the drying and wetting processes in the porous network that is composed of cylinders with radius r; r_m is the meniscus radius at the air-water interface (Do. [19]).

contained in pores takes place through the formation of cylindrical menisci along the pore walls, as presented in figure 4.

3.2 Experimental method

The soil water retention curve can be evaluated through the measurement of the matric suction using an oedometer cell (Fredlund and Rahardjo. [24]; Delage and Cui. [16]). This is a method that allows determining low water pressures, near the atmospheric pressure. If a saturated porous ceramic having pores thinner than those of soil is used, then the water phase continuity with the unsaturated soil is ascertained until the water phase in the soil specimen under consideration becomes discontinuous. It should be mentioned that the maximum suction that can be estimated is controlled by the air-entry pressure of the porous ceramic. In the present study, this pressure is set at 50 kPa. The highest soil water pressure head is approximately equal to 10 kPa (1 m of water column); this value is quite suitable for the study of the water retention characteristics of most sands. Moreover, it is ought to be noted that, during the test, no loading is applied. In this case, the use of the oedometer cell is highly encouraged by the fact that it is easy to install the porous ceramic that was provided for the cell.

A schematic view of the test apparatus is clearly illustrated in figure 5. This system consists of an odometer cell that has a diameter of 70 mm and height of 37 mm, supplied with a porous ceramic, and a graduated tube with a 7-mm inner diameter connected to a reservoir of water (Feia et al. [25]). As for the specimen, it is 27 mm thick and was prepared by compaction, in three layers, under dry conditions. It should be specified that the thickness was chosen smaller than the height of the odometer cell for the purpose of minimizing the potential errors that are associated with the matric suction along the specimen height. This would obviously introduce an uncertainty linked to both measurement and specimen thickness. An error analysis related to this uncertainty is presented later in the paper. Note also that diverse preparation methods, such as the pluviation method, can be used as well in order to achieve the needed relative density. The cell was then installed in a room where the temperature was set at 20°C, and the sample was wrapped in a film of cellophane in order to limit water evaporation. Since the preparation of the sample was done under dry conditions, the first step of the experiment started with the wetting path. Once the maximum suction level was attained, a gradual suction reduction was performed and the associated water content variations were measured and recorded. It is interesting to note that suctioning was done by positioning the level of water in the graduated tube lower than the sample center, as presented in Figure 5. Once the equilibrium was reached for the sands under study, after about 30 min, it was observed that the level of water inside the graduated tube changed. This made

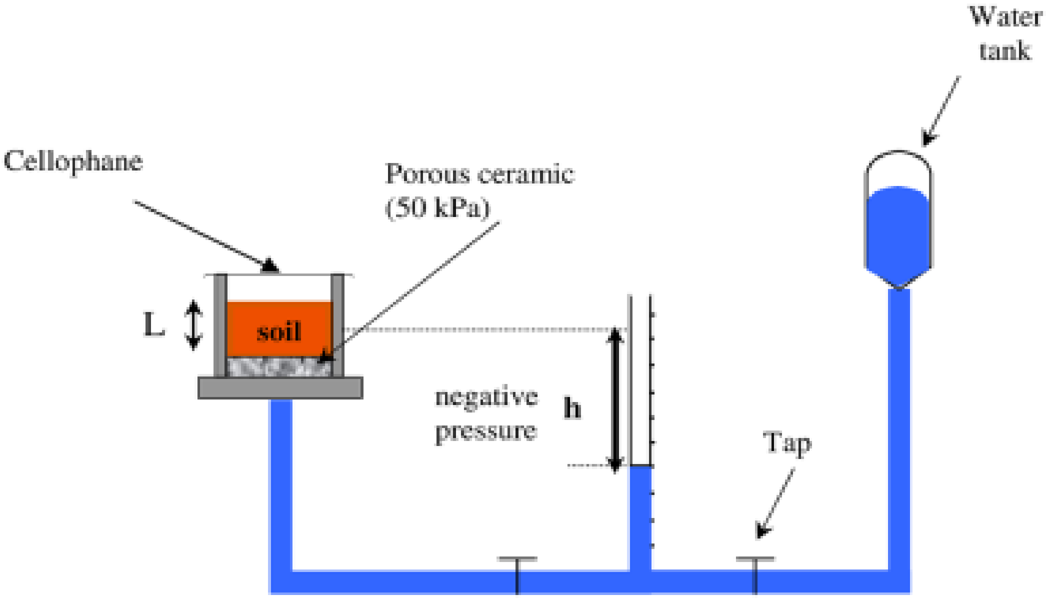


Figure 5: Schematic representation of the tensiometric method for the measurement of suction (Feia et al. [25]).

it possible to calculate the suction applied in each step by considering the vertical separation between the water level inside the graduated tube and the sample center. It should be noted that the reading accuracy for the water level in the tube was 1 mm, which represents a water volume of 38 mm³. This volume is satisfactorily smaller than that of the pore volume of the sample. In addition, the impact of the suction distribution along the height of the sample can be considered as negligible because the thickness of the specimen (27 mm) is relatively small. Bearing in mind the small area of the tube, one can say that the amount of water that was evaporated during the *time* required to *reach equilibrium was* also negligible. During the last phase of the wetting path, the water level in the graduated tube was positioned at the center of the sample after equilibrium; then, the drying path was applied by gradually increasing the suction to eventually reach the maximum level. It is worth noting that the drying path can be maintained uninterrupted to a point where the water phase is not continuous anymore. Consequently, the water content will not be reduced even when the suction is increased further. Therefore, the water retention curves for the wetting and drying paths can be derived through the application of the aforementioned approach using several steps of suction for each path.

Based on the water-retention curve, the Young-Laplace law can therefore be employed to estimate the pore size corresponding to each suction level. This law gives the suction as a function of the pore-access diameter as

$$s = u_a - u_w = \frac{4\sigma_s cos\theta}{D_p} \tag{6}$$

where *s* is the suction, D_p is the pore-access diameter, u_a and u_w are, respectively, the air and water pressures. Also, σ_s= 72.75 x 10^{-3} N x m^{-1} at 20°C refers to the tension at the surface between water and air, and θ is the angle of contact between the meniscus and solid. Note that θ depends on the solid mineral, surface cleanliness and roughness, menisci's movement (advancing or receding), and viscous effects (Decker et al. [17]; Espinoza and Santamarina [21]; Lourenço et al. [40]).

In the literature, the approximation cosθ ≈ 1 is a commonly accepted supposition for the estimation of the pore-size distribution (Innocentini and Pandolfelli. [32]; Mitchell and Soga. [43]; Hao et al. [29]).

3.3 Introduction of the experimental data

The model proposed here is applied to the experimental results of the tensiometric tests carried out by Feia et al. [25]. It was therefore decided to select three tests that were performed at different density indexes, i.e., 0.5, 0.7, and 0.9 (Figure 6).

It ought to be noted here that the density index is expressed by the equation (7) as

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \tag{7}$$

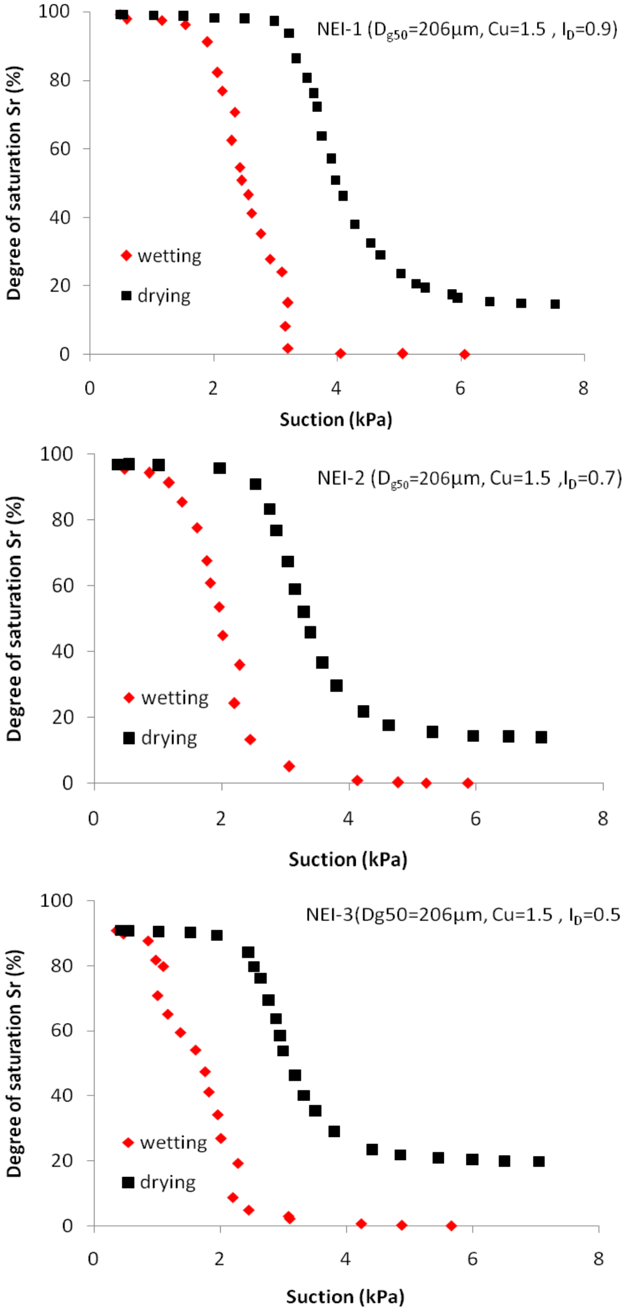


Figure 6: Experimental results used in this study (Feia et al. [25]).

Table 1: Characteristics of the used sands.

Sand	d_{g50} (µm)	C_u	e_{min}	e_{max}	ρ_s(t/m³)
NE34	206	1.5	0.557	0.884	2.65
Type of sand		**NEI-1**		**NEI-2**	**NEI-3**
Density index I_D		0.9		0.7	0.5

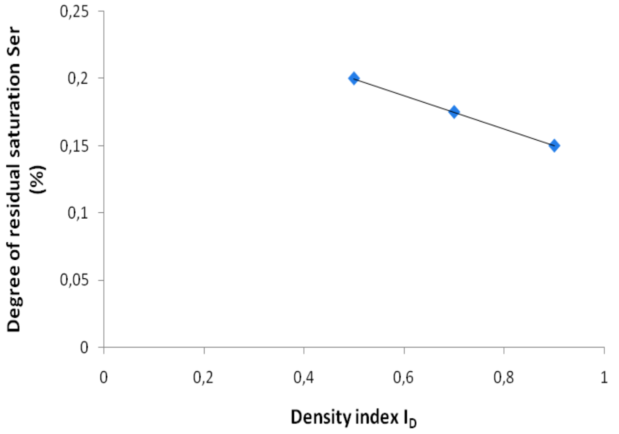


Figure 7: Variation of the degree of residual saturation as a function of the density index.

In the course of a tensiometric test, and more exactly during the drying path, a certain quantity of water still remains inside the sample. This amount of water that persists within the specimen, which corresponds to the degree of residual saturation, is mainly attributed to the water discontinuity between the tube and the porous medium. In addition, as reported in the literature, the value of the degree of residual saturation depends on several physical parameters of soil. One of these parameters is the density index I_D.

Figure 7 depicts the evolution of the degree of residual saturation against the density index, based on the data obtained from the tensiometric tests previously performed by Feia et al. [25]. One may easily note that this parameter varies linearly in accordance with the relationship expressed by equation (8) that is given below as

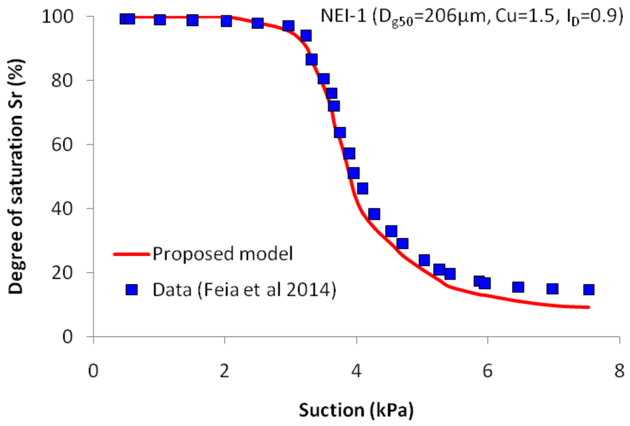
$$S_{er} = 0.12\ I_D + 0.26 \tag{8}$$

Equation (8) is in the form: y = ax + b
Hence, the proposed model can be expressed as follows:

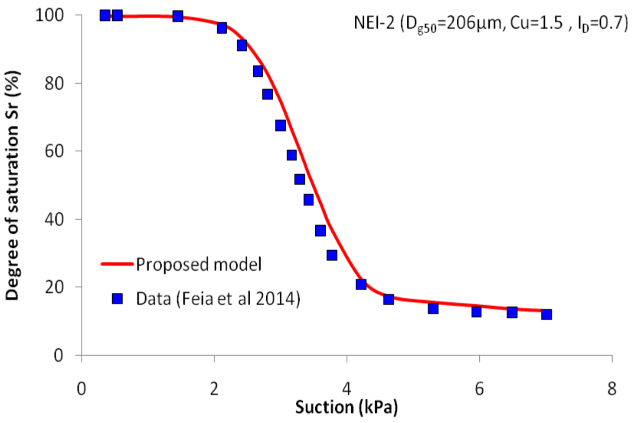
$$S_e = \left(1 + \left(\frac{s^*}{\alpha^*}\right)^n\right)^{\frac{1}{n}-1} - (0.12 I_D + 0.26) \tag{9}$$

4 Calibration of the model using the water retention curves

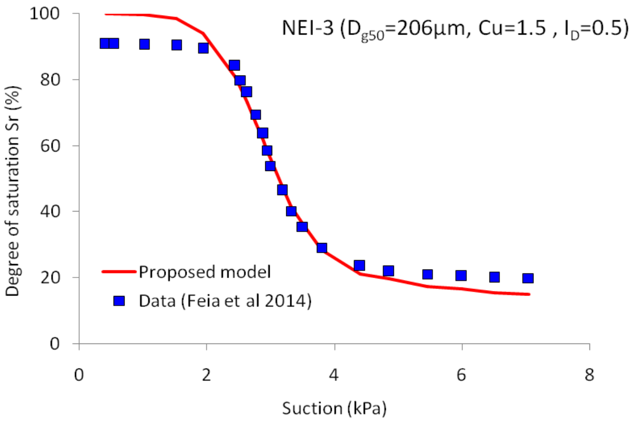
The water retention curve of the drying path was used to calibrate the model. Figure 8 shows clearly the calibration



a) Sand NEI-1



b) Sand NEI-2



c) Sand NEI-3

Figure 8: Calibration of the model on the basis of the experimental data for **a)** Sand NEI-1, **b)** Sand NEI-2, and **c)** Sand NEI-3

results. One may easily see that the model is in good agreement with the experimental data through the fitting of only two parameters (α and *n*).

Table 2: Values of the parameters of the proposed model for the three types of sand.

Sand / Model parameters	NEI-1 I_D= 0.9	NEI-2 I_D= 0.7	NEI-3 I_D= 0.5
α	4.5	3.4	3
n	8.5	7.3	6

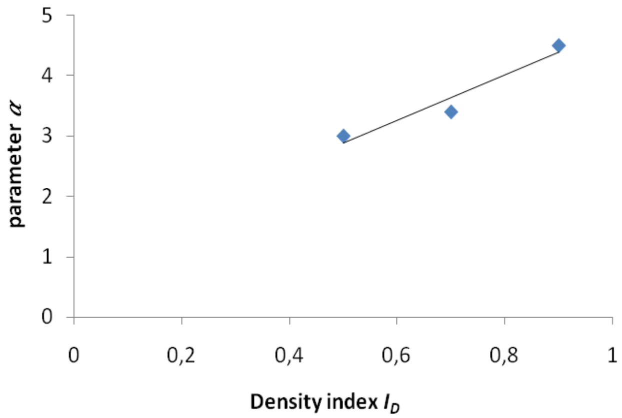


Figure 9: Evolution of the parameter α as a function of the density index I_{D}.

Also, the figure 8 depicts the calibration results that turned out to be quite consistent with the experimental data obtained from the tests. The findings indicate that the proposed model provides a much better prediction of the water retention curve in comparison with those found in the literature. Table 2 summarizes the parameters of the model.

Regarding figure 9, it presents the variation of the parameter α against the density index I_D. One can easily observe that the evolution curve tends to follow a linear form, i.e., y = ax+b. The overall trend of these data shows that this curve evolves in accordance with the equation given below

$$\alpha = 3.75 I_D + 1 \tag{10}$$

Furthermore, Figure 10, which displays the evolution of the parameter *n* as a function of the density index, shows that the evolution of the curve follows a linear form (y = ax+b). Moreover, the general trend of these data shows that this curve evolves in accordance with the equation given below

$$n = 6.25 I_D + 2.89 \tag{11}$$

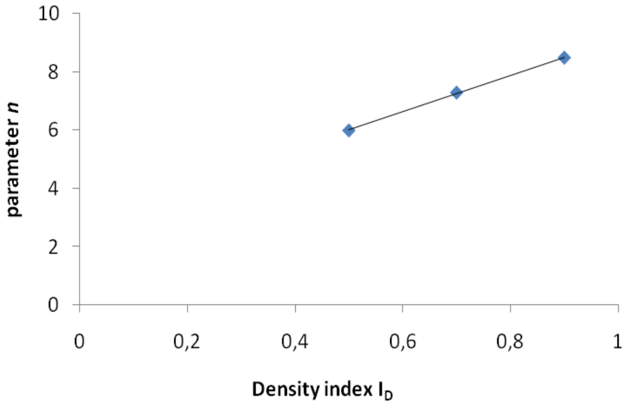


Figure 10: Evolution of the parameter (*n*) as a function of the density index $I_{D.}$

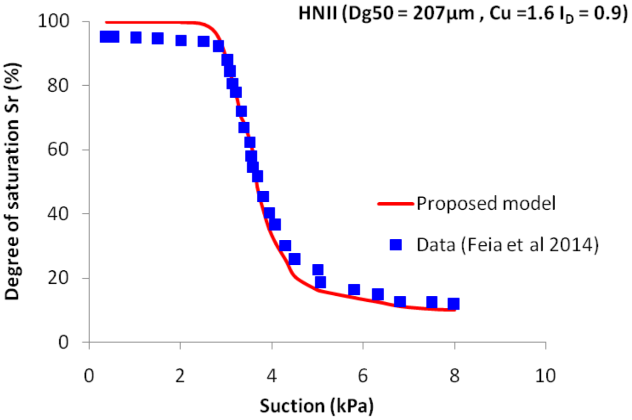


Figure 11: Validation of the model through the simulation of a test, with a density index I_D=0.9 and uniformity coefficient C_u=1.6.

As is clearly suggested in Figures (9) and (10), the values of α and n both increase as the density index I_D goes up.

4.1 Validation of the model

Once the calibration process was completed, the proposed model was then validated through the simulation of a test that was not used during calibration. To do this, it was decided to carry out a water retention test with soil type HN34 that had a uniformity coefficient C_U = 1.6 and a density index I_D=0.9. Based on the calibration results presented in figure (8), and according to the data presented in figures (9 and 10), it can be stated that parameters n and α both increased as the density index I_D augmented. Then, using the linear interpolation, presented by formulas (10) and (11), it was found that n = 8.51 and α=4.37, $\alpha^* = \alpha \times Cu$ =6.992. In addition, the simulation results displayed in figure (11) were then compared with the test results. This simple comparison showed that the simulation results were in good agreement with the test results, which validates the proposed model.

5 Application of the model to granular materials

5.1 Characteristics of the analyzed materials

The model was applied to four types of sand possessing the characteristics illustrated in Table 3 and represented by photo 1:

Photo 1: Four types of sand.

Table 3: Characteristics of the materials to be analyzed.

Sand	D_{50} (mm)	C_u	e_{min}	e_{max}	ρ_s (g/cm^3)
Type 1	0.18	1.5	0.51	0.79	2.65
Type 2	0.37	2.85	0.47	0.75	2.65
Type 3	0.42	2.47	0.47	0.76	2.65
Type 4	0.5	5	0.44	0.77	2.65

The grain-size distributions of all four types of sand obtained by sieve analysis are shown in figure 12 below:

5.2 Water retention curves

Figure 13 presents the water retention curves of all four types of sand, obtained by applying the proposed model

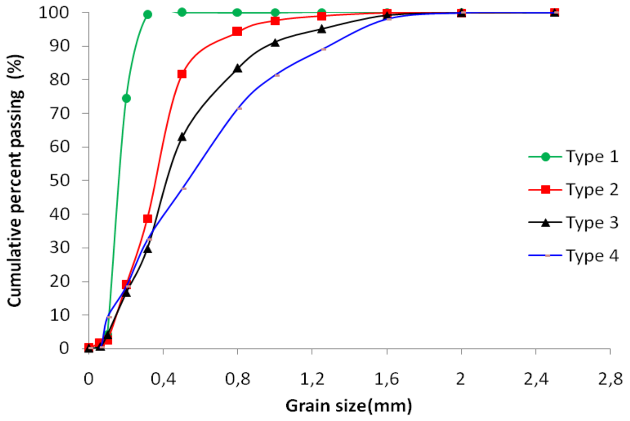


Figure 12: Grain-size distributions of all four sands obtained by sieve analysis.

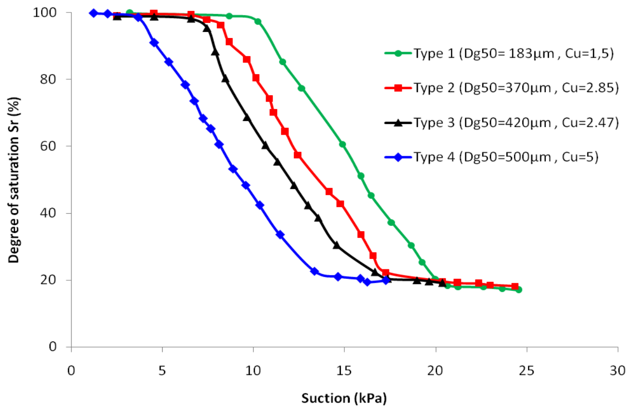


Figure 13: Water retention curves for all four types of sand.

for a density index equal to 0.90. It was noted that the degree of saturation started to diminish as the soil suction increased. One can clearly see that the different water retention properties of unsaturated soils are explicitly visible on these curves.

5.3 Effect of density index

Figure 14 shows the water retention curves for type 3 sand, obtained by applying the proposed model for different density index values. This same figure shows a comparison between the water retention curves of sand of type 3 for four different density indices (0.3, 0.50, 0.70, and 0.90). The results revealed that the suction increased as the density index went up, which is in very good agreement with the experimental findings. In addition, it is clearly noted that the different water retention properties of unsaturated soils can be observed on these curves.

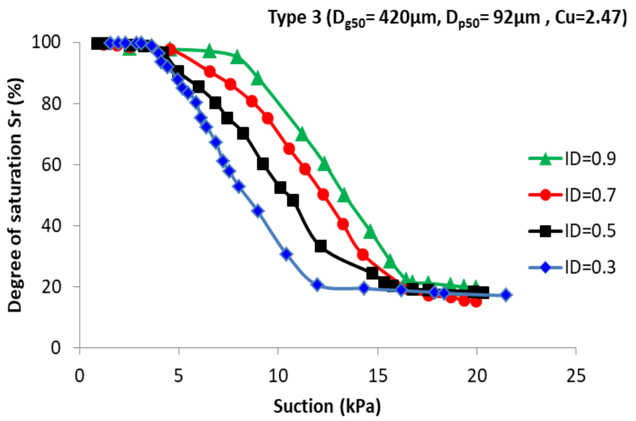


Figure 14: Water retention curves for type 3 sand for different density index values.

5.4 Pore-access size distribution of sands

Similarly, figure 15 shows a typical result of the pore-access size distribution curves for the four types of sand with a density index I_D = 0.50. A simple comparison between these pore-access size distribution curves and the particle size distribution curves indicates that the uniformity of the particle size curves induces a good uniformity of the pore-access size distribution curves **where D_{p50} is the diameter of the pores at 50% degree of saturation obtained from the results obtained by estimating the size of the pores using the model proposed.**

And the pore-access size distribution curves were obtained by the application of young-Laplace's law where the diameter of the pore access size can be estimated for each suction value. This law gives the suction as a function of the pore-access diameter as:

$$s = u_a - u_w = \frac{4\sigma_s cos\theta}{D_p}$$

Then

$$D_p = \frac{4\sigma_s cos\theta}{s}$$

5.5 Effect of density index

Figure 16 compares the pore-access size distributions for type 3 sand, with four different density indices (0.3, 0.50, 0.70, and 0.90). A very good consistency was then found between the pore-access size distributions and the density indexes. Moreover, it was found that the looser the sand, the larger the median pore size. It was also revealed that

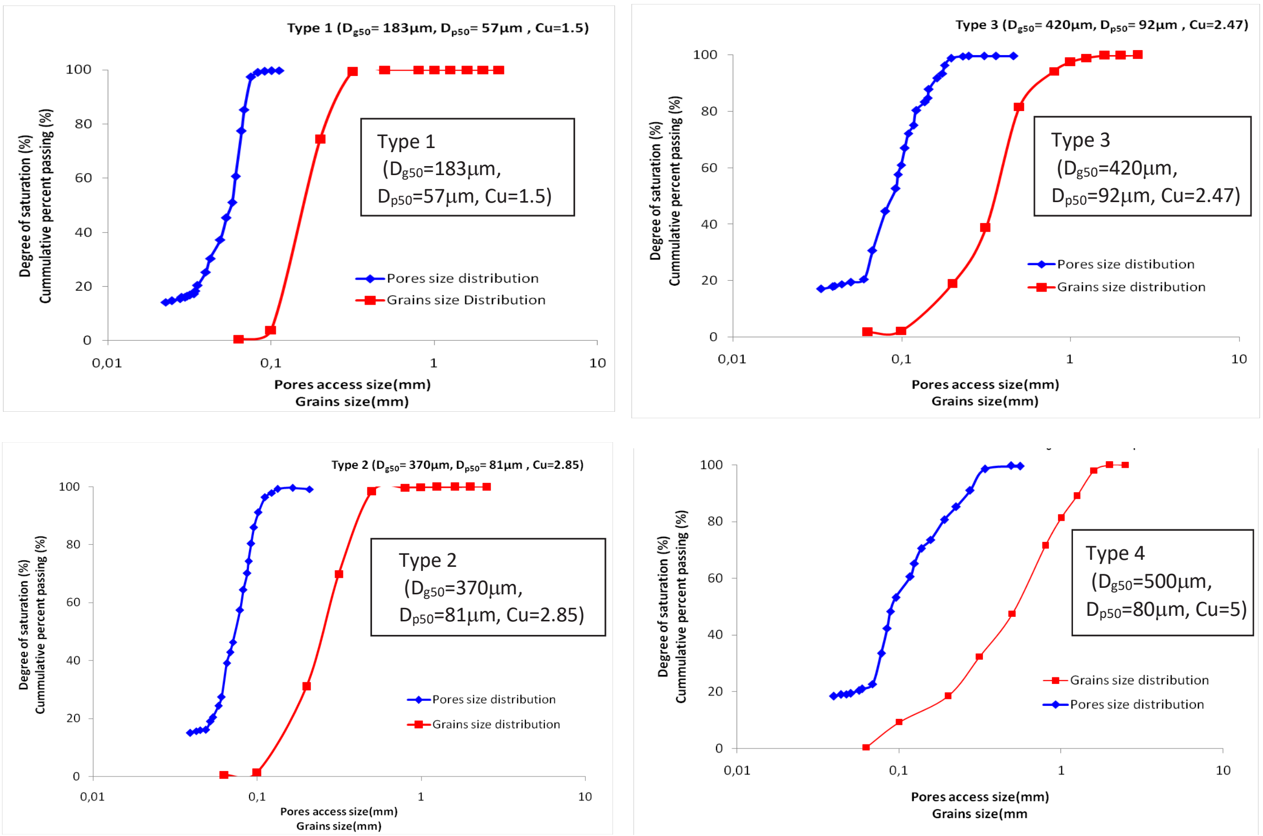


Figure 15: Pore-access size distributions for all four types of sand.

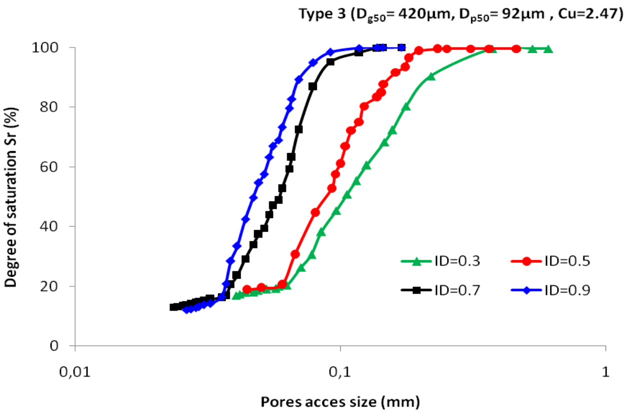


Figure 16: Effect of sand density index on pore-access size distribution.

the pore-access size distribution is also more spread out at lower density index values.

It was shown that for loose sand, the pore size becomes larger, and therefore, larger suctions are required to desaturate the pores. However, sands with higher density index values (dense sand) generally have finer pores.

Likewise, figure 17 shows a comparison between the pore sizes of sands with different particle distributions.

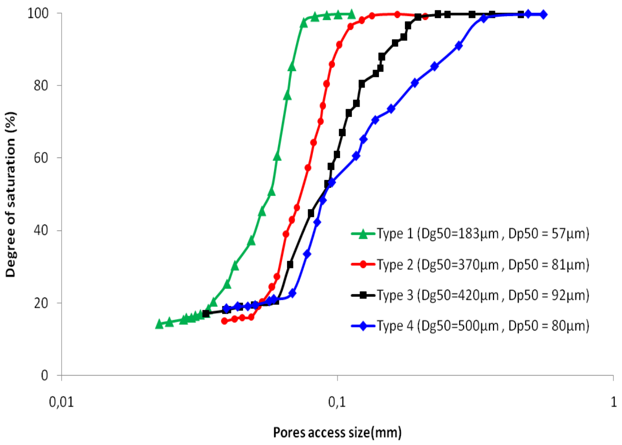
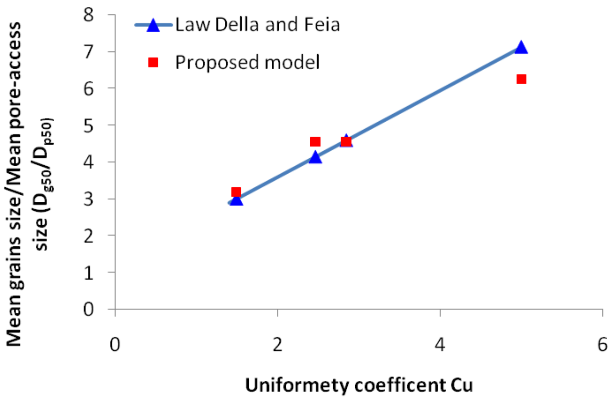
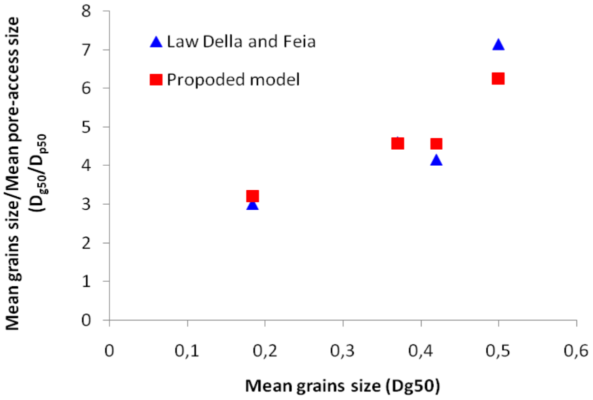


Figure 17: Comparison between the pore-access sizes of four types of sand.

It was observed that when the particle size becomes finer (the mean particle size D_{g50} is smaller), the pore size becomes smaller, and therefore, a direct relationship is found between D_{g50} (D_{g50} mean grain size) and D_{p50} (D_{p50} mean pore size). In addition, the pore size diversity increases as the particle size grows.



a) Variation of the (mean grains size /mean pore-access size) ratio as a function of uniformity coefficient



b) Variation of the mean grains size/mean pore-access size ratio as a function of the mean grains size (D_{g50})

Figure 18: Comparison between the results obtained by the proposed model and those calculated by the law of Della and Feia [47].

6 Comparison between the results of the proposed model and those obtained by the law of Della and Feia [47]

In this context, Figure 18 presents the variation of the (mean grains size over the pore-access size) ratio as a function of the uniformity coefficient C_u. The data used for this curve, which were obtained from our model, were then compared with those calculated using the aforementioned law of Della and Feia [47], found in the literature.

The proposed model allows to deduce the relationship between the ratio Dg_{50}/Dp_{50} and the Cu coefficient, where the used D_{g50} is defined as the grain diameter value at 50% of soil, and D_{p50} which is the pore access diameter at 50% of saturation obtained by the proposed model for the four soil types. Considering the Cu value for each soil, the D_{g50}/D_{p50} ratio of the proposed model can be easily obtained by Della and Feia law [47]. Thus, the values of D_{p50} are obtained for the four soil types, and the empirical D_{g50}/D_{p50} ratio can be deduced.

It should be noted that the mean grains size over the pore-access size ratio increased as the uniformity coefficient augmented. This ratio is well described by the expression that was established by Della and Feia [47] as

$$\frac{D_{g50}}{D_{p50}} = 1.18C_u + 1.24 \tag{12}$$

Figure 18 shows explicitly that the results are in very good agreement, which confirms the relevance of the law established by Della and Feia [47]

7 Conclusion

The purpose of this work was to develop a model that can be used to predict the water retention curve for granular materials. Satisfactory results were obtained with low uncertainty. These findings allowed drawing some interesting conclusions. The most essential ones are summarized as follows:

- The newly proposed analytical model can be used to predict the water retention curves of a granular material, like sandy soils. It should be noted that the proposed model equation is based on the physical properties of soil. This equation made it possible to describe the water retention curve and to estimate the pore-access size distribution without performing any experimental tests.
- In order to analyze the data in an efficient and reliable manner, only one criterion, i.e., the particle size distribution curve, was considered for the selection of soils. This curve can be used to derive the values of D_{50} and C_U.
- The water retention curves of soils (Feia et al. [25]) were fitted by the proposed model using the least squares method, which allowed determining the parameters α and n. Nevertheless, these parameters (n and α) could also be derived from the water retention curve while taking into account the effect of the density index I_D.

- The shape of the water retention curves obtained by the proposed model is similar to that of the curves provided in the literature, for similar materials. This model made it possible to draw the water retention curves of soil, without performing any experimental tests. In addition, the proposed model allowed plotting the pore-access size distribution curves without carrying out any additional experimental testing.
- It was observed that the means grains size/mean pore-access size ratio increased as the uniformity coefficient went up. Moreover, the relation established from the experimental study of Della and Feia [47] could be fitted to the data obtained by the proposed model. Therefore, the results found in this study are in agreement with those reported in the literature, which confirms the relevance of the law recently proposed by Della and Feia [47].

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