Studia Geotechnica et Mechanica, Vol. XXX, No. 3–4, 2008

SEEPAGE THROUGH DAM AND DEFORMABLE SOIL MEDIUM WITH CONSOLIDATION

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Abstract: The two-dimensional problem of the consolidation of an earth dam–subsoil system under body forces is presented. A certain loading scenario of the action (in proper time instants) of the subsoil's, the dam's and, subsequently, the reservoir water's deadweight and the seepage flow of groundwater arising thereof was assumed. Calculations for the Biot–Darcy poroelasticity model were performed using the finite element method. The results are presented as displacement and seepage velocity fields at selected moments in time. Also the possibility of critical water velocity gradients arising, which would lead to the loss of seepage stability by the earth dam–subsoil system, was analysed.

Streszczenie: Przedstawiono dwuwymiarowe zagadnienie procesu konsolidacji układu zapora ziemna–podłoże gruntowe pod działaniem sił masowych. W obliczeniach założono pewien scenariusz obciążenia wynikający z działania (w określonych przedziałach czasu) ciężaru własnego podłoża, zapory, a następnie ciężaru własnego wody w zbiorniku i przepływu filtracyjnego wody gruntowej. Obliczenia przeprowadzono metodą elementów skończonych dla modelu porosprężystości Biota. Wyniki obliczeń przedstawiono w postaci graficznej pól przemieszczeń oraz prędkości filtracji w wybranych momentach czasowych. W pracy przeanalizowano również możliwość pojawienia się krytycznych gradientów prędkości wody gruntowej, które mogą być przyczyną utraty stabilności filtracyjnej układu zapora ziemna–podłoże.

Резюме: Представлен двухразмерный вопрос процесса консолидации системы земляная плотина–грунтовое основание под действием массовых сил. В расчетах был предположен некоторый сценарий нагрузки, вытекающий из действия (в определенных пределах времени) собственного веса основания, плотины, а затем собственного веса воды в бассейне и фильтрационного протекания грунтовых вод. Расчеты были проведены методом конечных элементов для модели пороупругости Биота. Результаты расчетов были представлены графически в форме полей перемещений, а также скорости фильтрации в избранных моментах времени. В работе проведен также анализ возможности появления критических градиентов скорости грунтовых вод, которая может быть причиной потери фильтрационной устойчивсти системы земляная плотина–основание.

1. INTRODUCTION

The analysis of the phenomena occurring in the subsoil as a result of the construction of an earth dam is of a great interest to researchers because of the complexity of the consolidation and seepage processes. It is also an important engineering problem since when the hydraulic gradient is high, the seepage transport forces may lead to piping and soil liquefaction and, consequently, to the loss of stability by the soil medium.

In engineering research, the seepage problem is usually solved independently of soil medium consolidation. The calculations are made assuming that the solid phase and the liquid phase are nondeformable. Thus the initial stress and deformation produced by the consolidation process are neglected. In the authors' opinion, the magnitude of such stresses may have an impact on the seepage processes and on the seepage stability of the earth dam–subsoil system.

Consolidation theory was introduced by BIOT [5], [6] and in the next decades it was developed by a number of authors. The analytical study of poroelasticity equations and the determination of related effective parameters were the subjects of many papers (AURIAULT [3], AURIAULT at al. [1], [2], BAUER at al. [4], HORNUNG [12], and KELLER [13]). The analytical description of the consolidation process has a very complex form and thus the solutions obtained (DERSKI [9], GASZYŃSKI and KOMANIECKA [11]) are inapplicable.

This paper presents the results of numerical calculations for the seepage of water through the foundation-earth dam soil which take into account the consolidation process. The aim is to demonstrate a relationship between the two above phenomena.

The calculations performed are based on the following assumptions:

- the problem is two-dimensional;
- a diphase medium, consists of an elastic porous skeleton and a poorly compressible Newtonian fluid filling the skeleton's pores;
- a porous medium is an isotropic homogeneous body; the skeleton deformations are small, hence the components of the deformation tensor ε_{ij} can be assumed to be linear;
- the stress σ_{ij} in the skeleton of porous medium will apply to the total cross-sectional surface area of the skeleton–fluid system, even though in reality one should deduct the area occupied by pores from this surface area;
- the term *fluid pore stress* σ, referring to the total surface area of the cross-section occupied by the skeleton and pores filled with the fluid, is introduced according to the relation

$$\sigma = -pf, \tag{1}$$

where p is the fluid effective pressure and f stands for the medium volumetric porosity (which is constant).

In order to take into account the initial value of soil medium consolidation and the fact that stresses arose in the soil medium, the following scenario of the subsequent actions of the loads produced by the deadweight of the subsoil, the dam and the reservoir water at the specified time intervals was assumed:

P h a s e 1. At the instant $t = 0$ the subsoil – a soil layer with the thickness d and the length s (figure 1) – begins to consolidate under its own deadweight (preconsolidation).

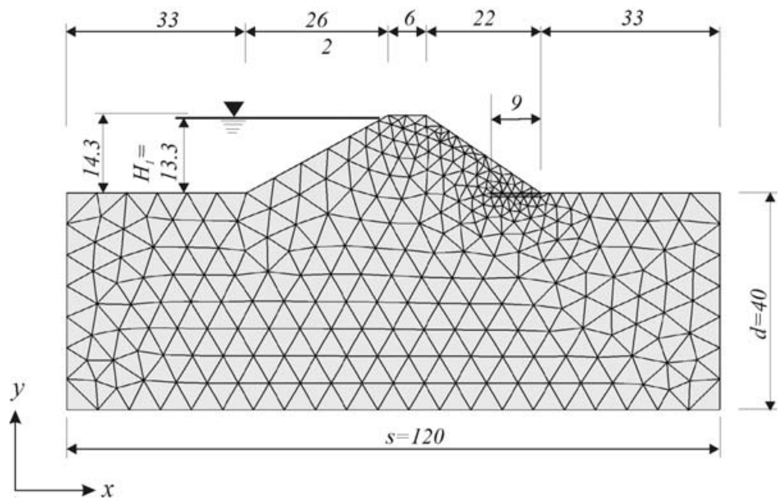

Fig. 1. The problem of geometry and initial finite element mesh

P h a s e 2. After $T_0 = 3.0 \cdot 10^9$ s $\approx$ 99 years the consolidation of the medium under its own deadweight ends and then the determined subsoil stress and strain are taken for further calculations. Thereafter at the instant T_0 the subsoil is loaded with an earth dam and the consolidation of the subsoil and the earth dam under the deadweight of the latter begins.

P h a s e 3. After the time $T_1 = 3.6 \cdot 10^9$ s $\approx$ 101 years the water reservoir is filled with water to the height H_1 above the ground level and consolidation begins due to the dam weight and the seepage transport forces resulting from the seepage flow of water through the reservoir and the subsoil. The calculations continue for a very long time (theoretically $t \to \infty$), which allows the process to become stable, the stress and strain in the dam–subsoil system to be determined, and the way of water seepage through the deformed (by the consolidation) porous medium to be shown.

There are some simplifications in the scenario: the time of earth dam construction and the time of reservoir filling are assumed to be immaterial to the ultimate consolidation of the dam–subsoil system, the Heaviside function $\eta(t - T_1)$ is used to describe the loading of the subsoil by the dam and the time of filling the reservoir to the height H_1 is assumed to be zero, which means that the application of stress in the fluid is also described by the Heaviside function.

The numerical solution presented demonstrates the new possibilities for analysing and calculating hydrotechnical structures, which take into account subsoil and earth dam consolidation. A similar finite element method approach to the geotechnical behaviour of an embankment on soft soils incorporating vertical drains was analysed during and after the construction period by BORGES [7].

In order to determine seepage with consolidation taken into account one must formulate a proper mathematical model and assume a method of solving it. Biot's poroelasticity model was adopted to describe the subsoil and earth dam consolidation process.

2. EQUATIONS OF CONSOLIDATION

A two-dimensional problem of poroelasticity under a plane state of stress was solved using the finite element method.

The system of the Biot poroelasticity equations, in a plane strain, boils down to a system of three differential equations which according to Biot's consolidation theory (STRZELECKI at al. [14]) have the form:

$$(M+2N)u_0\frac{\partial^2 u}{\partial x^2}+Nu_0\frac{\partial^2 u}{\partial y^2}+(M+N)v_0\frac{\partial^2 v}{\partial x\partial y}=-\frac{H}{R}\sigma_0\frac{\partial\sigma}{\partial x},$$

$$(M+2N)v_0\frac{\partial^2 v}{\partial x^2}+Nv_0\frac{\partial^2 v}{\partial y^2}+(M+N)u_0\frac{\partial^2 u}{\partial x\partial y}-\rho_s g\cdot\eta(t)=-\frac{H}{R}\sigma_0\frac{\partial\sigma}{\partial y},\tag{2}$$

$$\sigma_0\nabla^2\sigma=\frac{1}{RK}\sigma_0\frac{\partial\sigma}{\partial t}-\frac{H}{RK}\left(u_0\frac{\partial^2 u}{\partial x\partial t}+v_0\frac{\partial^2 v}{\partial y\partial t}\right).$$

The above system of the Biot poroelasticity equations for displacement is completed with constitutive relations for the Biot body, which for isothermal processes have the following forms:

- for stresses in the skeleton (effective stresses):

$$\sigma_{xx}=(2N+M)\varepsilon_{xx}+M\varepsilon_{yy}+\frac{Q}{R}\sigma,\qquad \sigma_{yy}=M\varepsilon_{xx}+(2N+M)\varepsilon_{yy}+\frac{Q}{R}\sigma,$$
$$\sigma_{zz}=(2N+M)\varepsilon+\frac{Q}{R}\sigma,\qquad \tau_{xy}=M\varepsilon_{xy};\tag{3}$$

- for fluid stress:

$$\sigma=Q\varepsilon+R\theta.\tag{4}$$

For the assumption adopted in the case of small deformations, the following linear soil medium deformation–displacement relations hold true:

$$\varepsilon_{xx}=\frac{\partial u}{\partial x},\quad \varepsilon_{yy}=\frac{\partial v}{\partial y},\quad \varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right),\quad \varepsilon=\varepsilon_{xx}+\varepsilon_{yy}.\tag{5}$$

Assuming the fluid to be poorly compressible we arrive at

$$\theta = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} . \tag{6}$$

For the purpose of calculation the hydraulic head h is introduced and expressed by the formula:

$$h = -\frac{\sigma - \sigma_a}{f\rho g} + y , \tag{7}$$

where:

M, N, R, H, Q	– the Biot constants of a medium,
$\rho_s g$	– the soil skeleton weight by volume with hydrostatic lift taken into account,
u, v	– the nondimensional values of displacement along the x- and y- axes,
σ	– the fluid stress in the pores,
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	– a normal stress along the x-, y- and z-axes, respectively,
τ	– the shear stress,
$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}, \varepsilon_{yx}$	– the components of deformation tensor,
ε	– the dilatation of skeleton,
θ	– the dilatation of fluid,
$\eta(t)$	– the Heaviside function effective changes in the load according to the scenario adopted,
$K = k/f^2\rho g$	– the permeability coefficient,
ρg	– the specific gravity of water,
k	– the Darcy coefficient of permeability,
U, V	– the fluid displacements along the x-axis and the y-axis, respectively,
h	– the hydraulic head,
$-\sigma_a/f$	– the atmospheric pressure.

The calculations of the consolidation process allow us to determine the evolution of the stress, strain and displacement status. Knowing the stress tensor one can calculate the principal stresses and knowing the internal friction angle and the cohesion one can determine the Mohr–Coulomb (plasticity) potential.

The calculation of the foregoing potential is very beneficial as it allows one to find out whether the stress is close to the plasticity limit. The limit value of the potential is equal to zero. A change in the sign of the potential in a given point of the area indicates that the stresses arising out of consolidation cause the limit state. In such a case, an elastoplasticity model should be applied. However, to the knowledge of the authors of this paper, such an elastoplasticity model (which would cover simultaneously seepage and consolidation processes) does not have any known limit solutions.

The Mohr–Coulomb (plasticity) potential is expressed by the formula:

$$MCP = \frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2}\sin\varphi - c\cos\varphi\,, \tag{8}$$

where:

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2}\,, \quad \sigma_2 = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2}$$

represent the principal stresses and φ and c stand for respectively the internal friction angle and cohesion.

3. NUMERICAL SOLUTION BY FINITE ELEMENT METHOD (FEM)

The following values of soil medium parameters were taken for the FEM calculations: the subsoil and dam coefficient of permeability: $k_1 = 1.5\cdot10^{-5}$ m/s, the horizontal drain coefficient of permeability: $k_2 = 10^{-4}$ m/s, the medium porosity: $f = 0.3$, the volumetric skeleton weight with hydrostatic lift taken into account: $\rho_s g = 1.4\cdot10^4$ N/m^3, the Biot constants based on Emmrich's laboratory tests (EMMRICH [10]): M = 433.3 MPa, N = 250 MPa, R = 150 MPa, H = 250 MPa, Q = 100 MPa, the internal friction angle: $\varphi = 25°$ and the cohesion: $c = 0.0$ Pa.

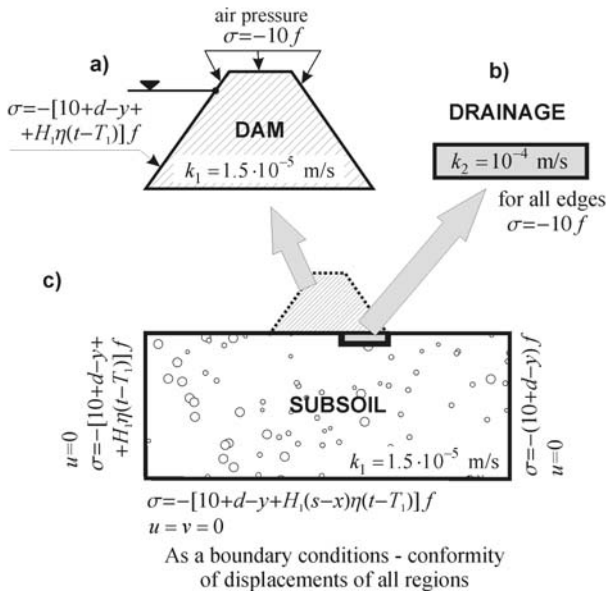

Fig. 2. Consolidation model:
a) dam boundary conditions, b) drain boundary conditions, c) subsoil boundary conditions

It was assumed that the thickness of the soil medium layer is 40 m and its span – 100 m and the ground water rise head after reservoir is filled amounts to H_1 = 13.2 m. The atmospheric pressure in the Biot equations (in accordance with COUSSY [8]) was assumed to be σ_a/f = 100 kPa.

The area geometry and the initial finite element mesh are shown in figure 1. The earth dam has the shape of a trapezium with a base of 54 m and a 6-m-wide crest. The dam is 14.3 m high. The slopes' inclination is 1:1.82 and 1:65. A 9-m-long and 0.8-m--deep horizontal drain is situated at the slope toe on the downstream side.

The consolidation area was divided into three separate regions as schematically shown in figure 2. The boundary conditions for region 1 (the earth dam's cross-section) are given in figure 2a, for region 2 (the horizontal drainage insert) in figure 2b and for region 3 (the subsoil) in figure 2c. It was assumed that the piezometric-pressure head is measured from the subsoil's lower edge.

The following boundary conditions were adopted:

$$u^{(0)} = 0, \quad v^{(0)} = 0, \quad \sigma^{(0)} = -f \cdot 10 \cdot (d - y) \quad [\text{kPa}],$$

where d is the depth (height) of the subsoil under the dam.

4. ANALYSIS OF CALCULATION RESULTS

Let us first consider the consolidation of the subsoil under its deadweight. In this case, it is best to characterize the consolidation through the seepage process resulting directly from subsoil stratum deformation.

4.1. SEEPAGE PROCESS DURING CONSOLIDATION OF SUBSOIL UNDER ITS DEADWEIGHT (PHASE 1)

Figure 3 shows the vector seepage velocity field for times t = 1 s, 10^3 s, 10^9 s. As one can see, in the course of consolidation the seepage process goes through several stages and the initial order of seepage velocity decreases from 10^{-5} m/s to 10^{-10} m/s after the time t = 10^3 s. After the consolidation, velocity vectors of seepage become zero.

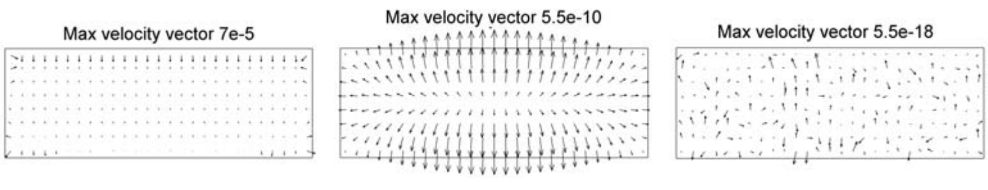

Fig. 3. Evolution of vector seepage field during consolidation of soil medium under its deadweight. From left : t = 1 s – the beginning of consolidation, t = 10^3 s, t = 10^9 s – the end of consolidation

4.2. SUBSOIL DISPLACEMENTS AND STRESSES UNDER ITS DEADWEIGHT (PHASE 2)

The distribution of soil displacements for $t = 10^9$ s versus stratum depth has a curvilinear character as shown in figure 4.

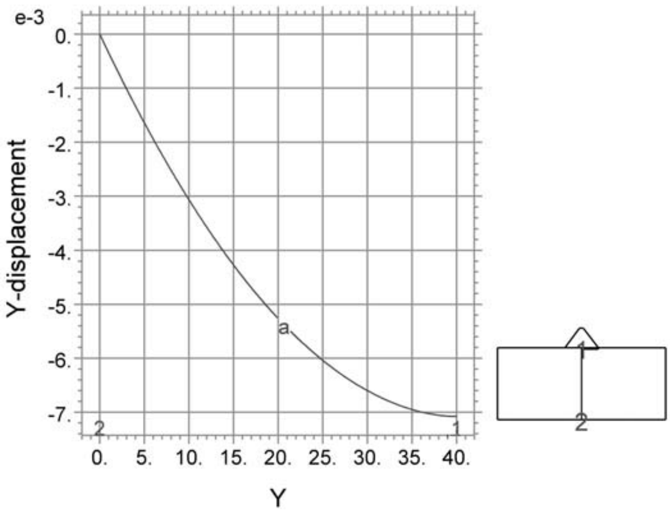

Fig. 4. Midspan vertical subsoil displacement curve generated by soil medium deadweight after the time $t = 10^9$ s

The same settlement of the upper substratum surface of $7 \cdot 10^{-3}$ m was obtained for the material constants adopted. The distribution of stress σ_{yy} is linear, and the highest compressive stress in the substratum floor amounts to $4.4 \cdot 10^2$ kN/m^2. Also the distribution of stress σ_{xx} is linear, and the highest compressive stress amounts to $2.7 \cdot 10^2$ kN. The Mohr–Coulomb potential is negative whereby no plasticization zone occurs under the adopted boundary condition parameters. Neither does a seepage stability loss hazard occur at any of the stages.

4.3. DISPLACEMENT AND STRESS DUE TO CONSOLIDATION UNDER DAM DEADWEIGHT AND THRUST OF WATER IN RESERVOIR (PHASE 3)

It is assumed that at the instant $T_0 = 3 \cdot 10^9$ s the consolidation of the subsoil and the dam under the deadweight of the latter begins and continues undisturbed (the reservoir remains empty) up to the instant $T_1 = 3.06 \cdot 10^9$ s, i.e. for about 2 years. At the moment T_1 the reservoir will be filled up to the set datum and from then on the deformation process is the result of both the dam deadweight and the thrust of the water in the reservoir. The displacements over the time after t = 3.03·10^{9} s are shown in figure 5.

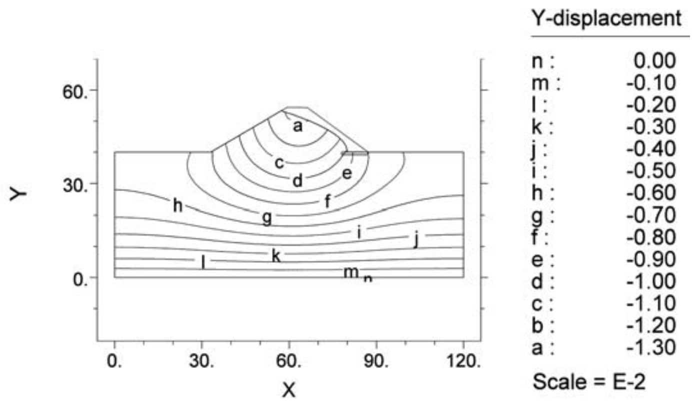

Fig. 5. Isolines of vertical displacement after the time $t = 3.03 \cdot 10^{9}$ s
(year after earth dam completion)

The calculated maximum subsidence is $-1.3 \cdot 10^{-2}$ m. The displacement diagram is symmetrical.

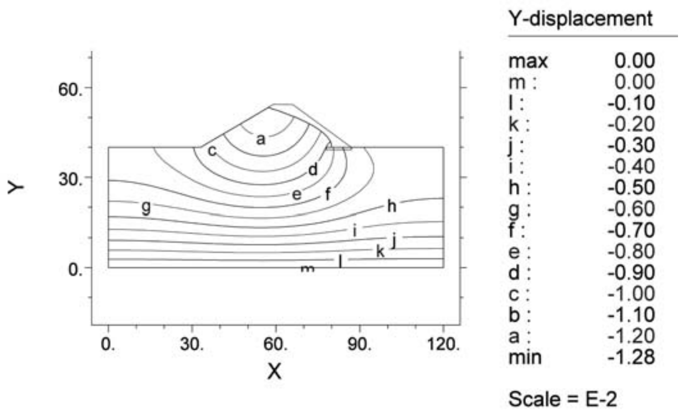

Fig. 6. Isolines of vertical displacement due to subsoil, dam and reservoir weight
after the time $t \rightarrow \infty$

The filling of the reservoir sets off additional deformations of the subsoil and the earth dam. Figure 6 shows the vertical displacements after the time $t = 10^{18}$ s; the latter is considered to be close to infinity and so it represents the ultimate consolidation process completion time. According to figure 6, water thrust and the seepage of water through the dam and the subsoil have a significant effect on the final shape of the displacement function. The maximum vertical displacement after the consolidation process ends amounts to $-1.28 \cdot 10^{-2}$ m and so it is smaller than the maximum displacement caused by the subsoil and earth dam deadweight. The picture of the deformed area is asymmetric. One can clearly see how significant effect the water reservoir has on the final form of the medium deformation.

The state of stress after the dam has been constructed and the reservoir filled with water is much more complex than the state of stress caused only by the medium deadweight.

4.4. SEEPAGE STABILITY AS ELEMENT OF BIOT'S BODY CONSOLIDATION

The seepage process with consolidation taken into account is described by the hydraulic head distribution, expressed by relation (7), and the seepage velocity field. The solution obtained allows one to analyse in detail the changes in the hydraulic head h under the deadweight of the subsoil, the earth dam and the water reservoir. The hydraulic head isolines after the completion of consolidation are shown in figure 7.

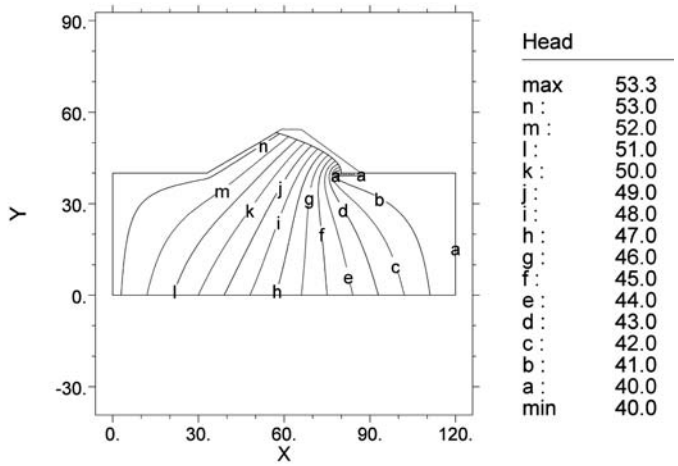

Fig. 7. Hydraulic head isolines after consolidation completion

According to figure 7, when the consolidation process is complete, the seepage boundary conditions and so the difference in the velocity potentials due to the difference between the water-facing side and downstream side water levels determine the hydraulic head function values. When the consolidation process is over, the changes in hydraulic head due to the medium deadweight which took place throughout the whole soil consolidation time interval are no longer observed.

In the consolidation process, one can find a critical velocity gradient at which seepage stability may be lost:

$$I_p \le I_{cr} = -(1-f)\frac{\rho_{os}}{\rho}, \tag{9}$$

where I_p is the vertical component of the hydraulic gradient in the direction opposite to the action of the gravitational forces.

If the limit hydraulic gradient is exceeded, piping resulting in soil liquefaction may occur. The latter may lead to soil outflow and a construction disaster. It is apparent that during the consolidation of the subsoil under its deadweight and the consolidation caused by the loading of the subsoil with the earth dam the seepage stability condition is not exceeded. Only as a result of the action of the water reservoir in the area of contact between the subsoil and the dam drainage system this condition is exceeded for the adopted subsoil parameters, as shown in figure 8.

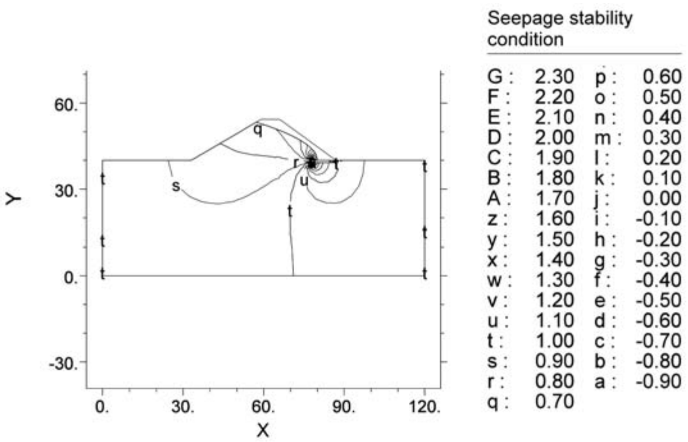

Fig. 8. Isolines of vertical hydraulic gradient component for $t \to \infty$ (seepage stability condition) within dam and subsoil under dam

Condition (9) is exceeded in the middle part of the dam (at a positive gradient) and so in the case of high grain-size diversity of the subsoil one can expect the washing out of finer soil particles (piping).

5. STRESS STATUS ANALYSIS BASED ON PLASTICITY POTENTIAL FUNCTION

The value of the plasticity potential is an indicator of the status of the area stress determined on the basis of the linear Biot model. If the limit value is reached, there is a risk of soil liquefaction under a particular construction. In the earth dam and subsoil example analyzed, the plasticity potential function was examined according to the scenario of Phases 2 and 3.

From the state of stress one can determine the Mohr–Coulomb potential, which for $t = 3.03\cdot10^9$ s is shown in figure 9. The area in which the potential is negative is the area in which the yield point has not been reached for the assumed Coulomb equation parameters φ and c. The plasticization area is bounded by the line g in figure 9. As one can see, the area covers the dam and the part of the subsoil under the dam. The highest positive values of the potential occur in the dam where the slope is in contact with the subsoil. The size of the area is determined by the adopted subsoil strength parameters. The values of φ and c indicate that the strength of soil medium is low whereby the determined plasticization zone is large.

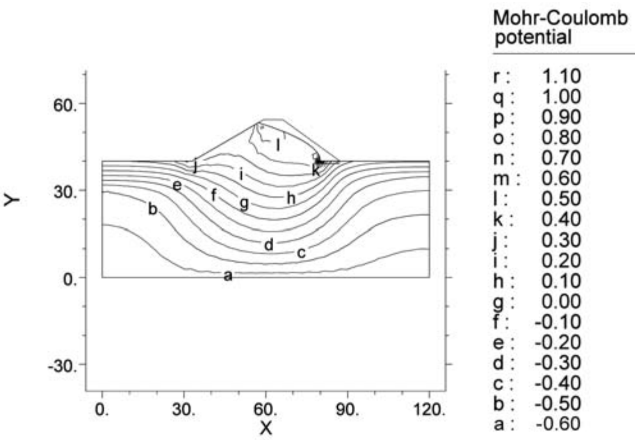

Fig. 9. The Mohr–Coulomb potential at the instant $t = 3.03\cdot10^9$ s

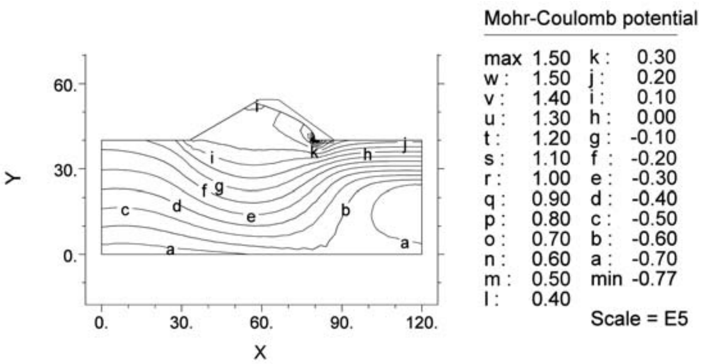

Fig. 10. The isolines of the Mohr–Coulomb potential (MCP) for $t \to \infty$

For Phase 3 the results of the calculation of the plasticity potential after the completion of the consolidation caused by the dam and water reservoir deadweight (T_∞ = 10^{18} s) are shown in figure 10. The plasticization area is bounded by the line h and covers the dam and the part of subsoil underneath. The *MCP* function reaches maximum values when the downstream slope comes into contact with the subsoil. It is apparent that due to the run-off pressure influence on the state of stress the dam on the downstream side is more exposed to stability loss.

It should be highlighted that the proposed use of *MCP* function is applicable only to the determination of the area where the elasticity limit has been reached. In order to calculate the stresses in the plasticity area, one should apply an appropriate elasto-plasticity model which includes the pressure distribution in the pores.

6. CONCLUSION

The presented numerical solution of the seepage process in relation to consolidation in accordance with a given scenario makes a detailed subsoil and earth dam analysis possible both at the stage of dam construction and after its completion.

A comparison of the numerical simulation results for the seepage of an incompressible fluid through a deformable medium with the results obtained for a nondeformable medium shows no significant discrepancies between the hydraulic head, water pressure or velocity field values. This means that for the linear Biot model applied the consolidation process (after appropriately long time) has no impact on the seepage problem solution. In other words, for the assumptions made in this paper it is possible to determine the seepage using the classic seepage flow theory based on Laplace's equation.

In the authors' opinion, the above is due to the fact that the physical and mechanical parameters of the medium are assumed to be invariable in the course of consolidation. Even porosity, which undoubtedly changes as a result of the dilatation of the skeleton, is assumed to be constant in the linearly elastic Biot model.

If some parameters (e.g., porosity) are made variable, this will certainly modify the seepage process and make it possible to take into account the effect of consolidation on the final seepage. However, the subsoil medium model becomes then nonlinear and the results may not be unique.

Thanks to the method of calculating the limit values of seepage velocity, as proposed in this paper, one can determine whether for given soil medium parameters there is a risk of piping and soil liquefaction and, consequently, the loss of stability by the soil medium. Furthermore, the proposed method of calculating the plasticity potential enables us to assess whether the dam and soil stress status is close to its limit in a particular area. The method allows us to determine the limit load value without a prior reference to the elastoplasticity model. The solutions obtained are the starting point for further research aimed at building a non-linear Biot model and undertaking validation experiments.

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