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THERMAL CONVECTION IN A FERROMAGNETIC FLUID SATURATING A POROUS MEDIUM

SUNIL*, PAVAN KUMAR BHARTI, DIVYA SHARMA

Department of Applied Sciences, National Institute of Technology (Deemed University), Hamirpur, (H.P.)–177 005, India.

Dedicated to late Professor R. C. Sharma

Abstract: This paper deals with the theoretical investigation of the thermal convection on a layer of ferromagnetic fluid heated from below saturating a porous medium subjected to a transverse uniform magnetic field. For a flat fluid layer contained between two free boundaries, an exact solution is obtained using a linear stability analysis theory and normal mode analysis method. In the case of stationary convection, the medium permeability and non-buoyancy magnetization both have a destabilizing effect on the system. The critical wave number and the critical magnetic thermal Rayleigh number for the onset of instability are also determined numerically for sufficiently large values of buoyancy magnetization parameter M_1 and the results are depicted graphically. The principle of the exchange of stabilities is found to hold true for the ferromagnetic fluid heated from below saturating a porous medium.

Streszczenie: Teoretycznie badano konwekcję termiczną w warstwie cieczy ferromagnetycznej ogrzewanej od dołu i wysycającej ośrodek porowaty poddany działaniu poprzecznego, jednorodnego pola magnetycznego. Otrzymano dokładne rozwiązanie dla płaskiej warstwy cieczy zawartej między dwoma swobodnymi brzegami, wykorzystując teorię liniowej stateczności i metodę trybu zwykłego. W przypadku ustalonej konwekcji zarówno przepuszczalność ośrodka, jak i niepowietrzna magnetyzacja wpływają destabilizująco na układ. Krytyczna liczba falowa i krytyczna magnetyczno-termiczna liczba Rayleigha na początku utraty stabilności są wyznaczone numerycznie dla wystarczająco dużych wartości parametru powietrznej magnetyzacji, a otrzymane wyniki zostały przedstawione graficznie. Stwierdzono, że zasada wymiany stabilności może być stosowana do cieczy ferromagnetycznej ogrzewanej od spodu i wysycającej ośrodek porowaty.

Резюме: Теоретически испытана термическая конвекция в слое ферромагнитной жидкости, нагреваемой снизу и насыщающей пористую среду, подверженную действию поперечного, гомогенного магнитного поля. Используя теорию линейной устойчивости и метод обыкновенного порядка, получено точное решение для плоского слоя жидкости, содержащейся между двумя произвольными берегами. в случае установленной конвекции как проницаемость среды, так и воздушная магнетизация дестабилизирующим образом влияет на систему. Критическое волновое число и критическое магнитно-термическое число Рейлея в начале потери устойчивости численно определены для достаточно больших значений параметра воздушной магнетизации, а полученные результаты были представлены графически. Было установлено, что принцип обмена устойчивости можно применять для ферромагнитной жидкости нагреваемого снизу и насыщающей пористую среду.

* To whom correspondence should be addressed.

1. INTRODUCTION

In the last millennium, the investigation on ferrofluids attracted attention of researchers because of their wider applications in area such as instrumentation, lubrication, vacuum technology, vibration damping, metals recovery, acoustics; their commercial usage includes vacuum feed-throughs for semiconductor manufacturing and related uses, pressure seals for compressors and blowers, engineering, medicine, chemical reactor and high-speed silent printers, etc. During the last half century, research on magnetic liquids has been very productive in many fields. The major perspectives are connected with a massive shocks and oscillation damping (earthquake, airbags), but the contemporary application concerned mostly seals and cooling of loudspeakers. Big efforts have been made to synthesize stable suspensions of magnetic particles with different performances in magnetism, fluid mechanics or physical chemistry. Many research workers have paid their attention towards the study of the applications of ferrofluid; see, for example, the investigations of MOSKOWITZ [1], HATHAWAY [2], BARCLAY [3], MORIMOTO et al. [4] and BAILEY [5]. Traditional ferrofluid products such as multistage rotary seals, exclusion seals, inertia dampers and loudspeakers allow now a well-established industry to be developed. Additionally, in the last few years, a number of new applications have emerged such as ferrofluid steppers, gauges and sensors (RAJ et al. [6]).

The monograph by ROSENWEIG [7] gives a detailed introduction to ferrohydrodynamics. FINLAYSON [8] has studied the convective instability of ferromagnetic fluids, whereas thermoconvective stability of ferrofluids without considering buoyancy effects has been investigated by LALAS and CARMI [9]. SCHWAB et al. [10] have investigated experimentally the Finlayson's problem in the case of a strong magnetic field and detected the onset of convection by plotting the Nusselt number versus the Rayleigh number. Then, the critical Rayleigh number corresponds to a discontinuity in the slop. Later, STILES and KAGAN [11] have examined the experimental problem reported by SCHWAB et al. [10] and generalized the Finlayson's model assuming that under a strong magnetic field, the rotational viscosity augments the shear viscosity. The theory of thermal convection of a non-magnetic fluid layer heated from below under varying assumptions of hydromagnetics has been treated in detail by CHANDRASEKHAR [12]. The Bénard convection in ferromagnetic fluids has been considered by many authors (RUDRAIAH and SHEKAR [13], ANISS et al. [14], SIDDHESHWAR [15], QIN and KALONI [16], SIDDHESHWAR [17], ZEBIB [18], SOUHAR et al. [19], ANISS et al. [20], SIDDHESHWAR and ABRAHAM [21]). More recently, SUNIL et al. [22]–[24] have studied the effect of dust particles and rotation on thermal convection in a ferromagnetic fluid. In all the above studies, the medium has been considered to be non-porous. A good formalism of the basic equations of a layer of fluid heated from below in porous medium has been provided by JOSEPH [25]. A comprehensive review of the literature concerning thermal convection in a fluidsaturated porous medium may be found in the book by NIELD and BEJAN [26] and references therein.

The thermoconvective instability in a ferromagnetic fluid saturating a porous medium of very large permeability subjected to a vertical magnetic field has been discussed by VAIDYANATHAN et al. [27] who using the Brinkman model have indicated that only stationary convection can exist. A porous medium of very low permeability allows us to use the Darcy's model. This is because for a medium of very large stable particle suspension, the permeability tends to be small justifying the use of Darcy's model. This is because in a medium of very large stable particle suspension, the permeability tends to be small justifying the use of Darcy's model. This is because the viscous drag force is neglibility small in comparison with the Darcy's resistance due to the large particle suspension. Darcy's law governs the flow of ferromagnetic fluid through an isotropic and homogeneous porous medium. However, to be mathematically compatible and physically consistent with Navier–Stokes equations, BRINKMAN [28], [29] heuristically proposed the introduction of the term $(\mu/\varepsilon)\nabla^2\mathbf{q}$ (now known as the Brinkman term) in addition to the Darcian term $-(\mu/k_1)\mathbf{q}$. But the main effect is through the Darcian term; the Brinkman term contributes a very little effect for flow through a porous medium. Therefore, Darcy's law is proposed heuristically to govern the flow of this ferromagnetic fluid saturating a porous medium.

The results of the paper, therefore, might become potentially interesting in metallurgy, semiconductor industry and geophysics. A layer of ferromagnetic fluid heated from below saturating a porous medium has relevance and importance in chemical technology, geophysics and bio-mechanics. The present paper, therefore, deals with the thermal convection in a ferromagnetic fluid in homogeneous and isotropic porous medium of very low permeability, allowing the use of Darcy's model, subjected to a vertical magnetic field. This problem, to the best of our knowledge, has not been investigated yet and we believe that the present study can serve as a theoretical support for an experimental investigation.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Here we consider an infinite, horizontal layer of the thickness d of an electrically non-conducting incompressible ferromagnetic fluid in porous medium, heated from below and that a uniform temperature gradient

$$\beta\left(=\left|\frac{dT}{dz}\right|\right)$$

is maintained (see figure 1). A uniform magnetic field H_0 acts along the vertical direction which is taken as the z-axis. The temperatures at the bottom and top surfaces $z = \mp 0.5d$ are T_0 and T_1, respectively. Both the boundaries are taken to be free and perfect conductors of heat. The gravity field $\mathbf{g} = (0, 0, -g)$ pervades the system. This ferromagnetic fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of the porosity ε and the medium permeability k_1, where the porosity is defined by

$$\varepsilon = \frac{\text{volume of the voids}}{\text{total volume}}, \quad (0 < \varepsilon < 1).$$

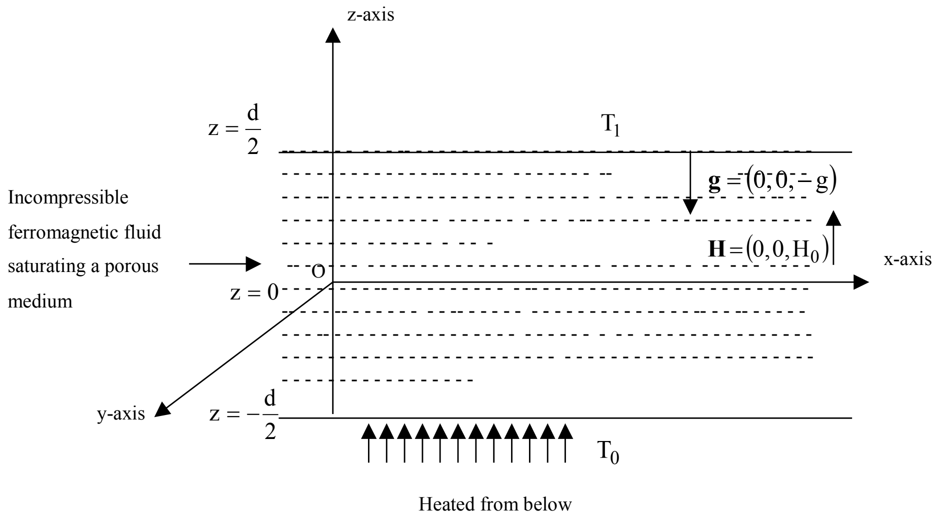

Fig. 1. Geometrical configuration

For very fluffy foam materials, ε is nearly one, and in beds of packed spheres ε is in the range of 0.25–0.50.

The mathematical equations governing the motion of a ferromagnetic fluid saturating a porous medium for the above model are as follows:

- The continuity equation for an incompressible fluid is

$$\nabla.\mathbf{q} = 0. \tag{1}$$

- The momentum equation for Darcy's model is

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \rho \mathbf{g} + \nabla.(\mathbf{HB}) - \frac{\mu}{k_1}\mathbf{q}. \tag{2}$$

- The temperature equation for an incompressible ferromagnetic fluid is

$$\left[\rho_0 C_{V,H} - \mu_0 \mathbf{H}.\left(\frac{\partial \mathbf{M}}{\partial T}\right)_{V,\mathbf{H}}\right]\frac{dT}{dt} + (1-\varepsilon)\rho_s C_s \frac{\partial T}{\partial t} + \mu_0 T\left(\frac{\partial \mathbf{M}}{\partial T}\right)_{V,\mathbf{H}} . \frac{d\mathbf{H}}{dt} = K_1 \nabla^2 T + \Phi . \tag{3}$$

• The density equation of state is

$$\rho = \rho_0[1-\alpha(T-T_a)] , \tag{4}$$

where ρ, ρ_0, $\mathbf{q}$, t, p, μ, μ_0, $\mathbf{H}$, $\mathbf{B}$, $C_{V,H}$, T, $\mathbf{M}$, K_1, α and Φ are the fluid density, reference density, Darcian (filter) velocity, time, pressure, dynamic viscosity (constant), permeability of free space, magnetic field intensity, magnetic induction, heat capacity at constant volume and magnetic field, temperature, magnetization, thermal conductivity, thermal expansion coefficient and viscous dissipation factor containing the second-order terms in velocity, respectively. Φ being second order small may be neglected. T_a is the average temperature given by

$$T_a = \frac{(T_0+T_1)}{2} ,$$

where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer. The partial derivatives of $\mathbf{M}$ are material properties, which can be evaluated once the magnetic equation of state, such as (8), is known. Two additional complications are assumed negligible in equation (2): we assume that the viscosity is isotropic and independent of magnetic field intensity. We also employ the Boussinesq approximation by allowing the density to change only in the gravitational body force term.

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, become

$$\nabla.\mathbf{B} = 0 , \quad \nabla\times\mathbf{H} = \mathbf{0} . \tag{5a, b}$$

In the Chu formulation of electrodynamics (PENFIELD & HAUS [30]), the magnetic field, magnetization and the magnetic induction are related by

$$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) . \tag{6}$$

We assume that the magnetization is aligned with the magnetic field, but allows a dependence on the magnitude of the magnetic field as well as the temperature

$$\mathbf{M} = \frac{\mathbf{H}}{H} M(H,T) . \tag{7}$$

The magnetic equation of state is linearized about the magnetic field H_0 and an average temperature T_a to become

$$M = M_0 + \chi(H-H_0) - K_2(T-T_a) , \tag{8}$$

where: H_0 is the uniform magnetic field intensity of the fluid layer when placed in an external magnetic field intensity $\mathbf{H} = H_0^{\mathrm{ext}}\hat{\mathbf{k}}$, $\chi = (\partial M/\partial H)_{H_0,T_a}$ is the magnetic susceptibility, $K_2 = -(\partial M/\partial T)_{H_0,T_a}$ is the pyromagnetic coefficient and $H = |\mathbf{H}|$, $M = |\mathbf{M}|$, $M_0 = M(H_0,T_a)$ and $\hat{\mathbf{k}}$ is unit vector in the z-direction.

The basic state is assumed to be quiescent state and is given by

$$\mathbf{q} = \mathbf{q}_b = \mathbf{0}\,,\quad p = p_b(z)\,,\quad \rho = \rho_b(z)\,,\quad T = T_b(z) = -\beta z + T_a\,,\quad \beta = \frac{T_0 - T_1}{d}\,,$$
$$\mathbf{H}_b = \left[H_0 - \frac{K_2 \beta z}{1+\chi}\right]\hat{\mathbf{k}}\,,\quad \mathbf{M}_b = \left[M_0 + \frac{K_2 \beta z}{1+\chi}\right]\hat{\mathbf{k}}\,,\quad H_0 + M_0 = H_0^{\text{ext}}\,, \tag{9}$$

where the subscript b denotes the basic state.

3. THE PERTURBATION EQUATIONS

We shall analyze the stability of the basic state by introducing the following perturbations:

$$\mathbf{q} = \mathbf{q}_b + \mathbf{q}'\,,\; p = p_b(z) + \delta p\,,\; T = T_b(z) + \theta\,,\; \mathbf{H} = \mathbf{H}_b(z) + \mathbf{H}'\,,\; \mathbf{M} = \mathbf{M}_b(z) + \mathbf{M}'\,, \tag{10}$$

where $\mathbf{q}' = (u, v, w)$, δp, θ, $\mathbf{H}' = (H_1', \mathrm{H}_2', \mathrm{H}_3')$ and $\mathbf{M}'$ are perturbations in velocity, pressure, temperature, magnetic field intensity and magnetization, respectively. These perturbations are assumed to be small and then the linearized perturbation equations become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\,, \tag{11}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p + \mu_0(M_0 + H_0)\frac{\partial H_1'}{\partial z} - \frac{\mu}{k_1}u\,, \tag{12}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p + \mu_0(M_0 + H_0)\frac{\partial H_2'}{\partial z} - \frac{\mu}{k_1}v\,, \tag{13}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p + \mu_0(M_0 + H_0)\frac{\partial H_3'}{\partial z} - \frac{\mu}{k_1}w - \mu_0 K_2 \beta H_3' + \frac{\mu_0 K_2^2 \beta \theta}{1+\chi} + g\alpha\rho_0\theta\,, \tag{14}$$

$$\rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 T_0 K_2 \varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \Phi'}{\partial z}\right) = K_1 \nabla^2 \theta + \left[\rho C_2 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{(1+\chi)}\right] w\,, \tag{15}$$

where $\rho C_1 = \varepsilon \rho_0 C_{V,H} + (1-\varepsilon)\rho_S C_S + \varepsilon \mu_0 K_2 H_0$, $\rho C_2 = \rho_0 C_{V,H} + \mu_0 K_2 H_0$,

$$\left.\begin{aligned} H_3' + M_3' &= (1+\chi)H_3' - K_2\theta, \\ H_i' + M_i' &= \left(1 + \frac{M_0}{H_0}\right)H_i' \quad (i = 1, 2), \end{aligned}\right\} \tag{16}$$

where we have assumed $K_2(T_b - T_a) << (1+\chi)H_0$. Thus the analysis is restricted to physical situation in which the magnetization induced by temperature variation is small compared to that induced by the external magnetic field. Equation (5b) means we can write $\mathbf{H}' = \nabla\Phi'$, where Φ' is the perturbed magnetic potential.

Eliminating *u*, *v*, δp from equations (12)–(14), using (11), we obtain

$$\left(\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t} + \frac{\mu}{k_1}\right)\nabla^2 w = -\mu_0 K_2 \beta\left(\nabla_1^2 \frac{\partial \Phi'}{\partial z}\right) + \rho_0 g\alpha(\nabla_1^2\theta) + \frac{\mu_0 K_2^2 \beta}{(1+\chi)}(\nabla_1^2\theta). \tag{17}$$

From (16), we have

$$(1+\chi)\frac{\partial^2 \Phi'}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right)\nabla_1^2 \Phi' - K_2 \frac{\partial \theta}{\partial z} = 0. \tag{18}$$

Analyzing the disturbances in normal modes, we assume that the perturbation quantities are of the form

$$(w, \theta, \Phi') = [W(z,t), \Theta(z,t), \Phi(z,t)]\exp i(k_x x + k_y y), \tag{19}$$

where k_x, k_y are the wave numbers along the *x*- and *y*-directions, respectively, and $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wave number.

Equations (17), (15) and (18), using equation (19), become

$$\left(\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t} + \frac{\mu}{k_1}\right)\left(\frac{\partial^2}{\partial z^2} - k^2\right)W = \frac{\mu_0 K_2 \beta}{1+\chi}\left[(1+\chi)\frac{\partial \Phi}{\partial z} - K_2\Theta\right]k^2 - \rho_0 g\alpha k^2 \Theta, \tag{20}$$

$$\rho C_1 \frac{\partial \Theta}{\partial t} - \mu_0 T_0 K_2 \varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \Phi}{\partial z}\right) = K_1\left(\frac{\partial^2}{\partial z^2} - k^2\right)\Theta + \left(\rho C_2 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{1+\chi}\right)W, \tag{21}$$

$$(1+\chi)\frac{\partial^2 \Phi}{\partial z^2} - \left(1 + \frac{M_0}{H_0}\right)k^2 \Phi - K_2 \frac{\partial \Theta}{\partial z} = 0. \tag{22}$$

Equations (20)–(22) give the following dimensionless equations

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t^*} + \frac{1}{k_1^*}\right)(D^2 - a^2)W^* = aR^{1/2}[M_1 D\Phi^* - (1+M_1)T^*], \tag{23}$$

$$P_r' \frac{\partial T^*}{\partial t^*} - \varepsilon P_r M_2 \frac{\partial}{\partial t^*}(D\Phi^*) = (D^2 - a^2)T^* + aR^{1/2}(1 - M_2)W^*, \tag{24}$$

$$D^2\Phi^* - a^2 M_3 \Phi^* - DT^* = 0, \tag{25}$$

where the following non-dimensional parameters are introduced:

$$t^* = \frac{\nu t}{d^2},\ W^* = \frac{Wd}{\nu},\ \Phi^* = \frac{(1+\chi)K_1 aR^{1/2}}{K_2 \rho C_2 \beta \nu d^2}\Phi,\ R = \frac{g\alpha\beta d^4 \rho C_2}{\nu K_1},\ T^* = \frac{K_1 aR^{1/2}}{\rho C_2 \beta \nu d}\Theta,$$

$$a = kd,\ z^* = \frac{z}{d},\ D = \frac{\partial}{\partial z^*},\ k_1^* = \frac{k_1}{d^2},\ P_r = \frac{\nu}{K_1}\rho C_2,\ P_r' = \frac{\nu}{K_1}\rho C_1,$$

$$M_1 = \frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha\rho_0 g},\quad M_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_2},\quad M_3 = \frac{\left(1 + \dfrac{M_0}{H_0}\right)}{(1+\chi)}.$$

4. EXACT SOLUTION FOR FREE BOUNDARIES

Here the simplest boundary conditions chosen, namely free-free, no-spin, isothermal with infinite magnetic susceptibility χ in the perturbed field, keep the problem analytically tractable and serve the purpose of providing a qualitative insight into the problem. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution, whose properties guide our analysis. Thus the exact solution of the system (23)–(25) subject to the boundary conditions

$$W^* = D^2 W^* = T^* = D\Phi^* = 0 \quad \text{at} \quad z = \pm\frac{1}{2} \tag{26}$$

is written in the form

$$\begin{aligned} &W^* = A_1 e^{\sigma t^*} \cos \pi z^*,\quad T^* = B_1 e^{\sigma t^*} \cos \pi z^*, \\ &D\Phi^* = C_1 e^{\sigma t^*} \cos \pi z^*,\quad \Phi^* = \left(\frac{C_1}{\pi}\right) e^{\sigma t^*} \sin \pi z^*, \end{aligned} \tag{27}$$

where A_1, B_1, C_1 are constants and σ is the growth rate which is, in general, a complex constant.

Substituting equations (27) in equations (23)–(25) and dropping asterisks for convenience, we arrive at the following equations

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{k_1}\right)(\pi^2+a^2)A_1+(aR^{1/2}M_1)C_1-aR^{1/2}(1+M_1)B_1=0\,, \tag{28}$$

$$(1-M_2)aR^{1/2}A_1-(\pi^2+a^2+P'_r\sigma)B_1+(\varepsilon P_r M_2\sigma)C_1=0\,, \tag{29}$$

$$-\pi^2B_1+(\pi^2+a^2M_3)C_1=0\,. \tag{30}$$

For existence of non-trivial solutions of the above equations, the determinant of the coefficients of A_1, B_1, C_1 in equations (28)–(30) must vanish. This determinant on simplification yields

$$V\sigma_i^2+iW\sigma_i+X=0\,, \tag{31}$$

where:

$$V=-\frac{(1+x)}{\varepsilon}\{(P'_r-\varepsilon P_r M_2)+xP'_r M_3\}\,,$$

$$W=(1+x)(1+xM_3)\left\{\frac{1}{\varepsilon}(1+x)+\frac{1}{P_\ell}P'_r\right\}-\frac{1}{P_\ell}(1+x)\varepsilon P_r M_2\,,$$

$$X=\frac{1}{P_\ell}(1+x)^2(1+xM_3)-R_1x(1-M_2)\{1+x(1+M_1)M_3\}\,,$$

where

$$R_1=\frac{R}{\pi^4}\,,\qquad x=\frac{a^2}{\pi^2}\,,\qquad i\sigma_i=\frac{\sigma}{\pi^2}\qquad\text{and}\qquad \mathrm{P}_\ell=\pi^2k_1\,.$$

5. DISCUSSION OF RESULTS

5.1. THE CASE OF STATIONARY CONVECTION

When the instability sets in as stationary convection in the case $M_2 \cong 0$, the marginal state will be characterized by $\sigma_i=0$ (CHANDRASEKHAR [12]), then the Rayleigh number is given by

$$R_1=\frac{(1+x)^2(1+xM_3)}{xP_\ell[1+xM_3(1+M_1)]}\,, \tag{32}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x, buoyancy magnetization parameter M_1, non-buoyancy magnetization parameter M_3 and medium permeability parameter P_ℓ (the Darcy number).

The classical result in respect of Newtonian fluids can be obtained as the limiting case of the present study.

Setting $M_3 = 0$ in equation (32), we get

$$R_1 = \frac{(1+x)^2}{xP_\ell}, \tag{33}$$

the classical Rayleigh–Bénard result in porous medium for the Newtonian fluid case.

To investigate the effects of medium permeability and non-buoyancy magnetization, we examine the behaviour of dR_1/dP_ℓ and dR_1/dM_3 analytically. Equation (32) yields

$$\frac{dR_1}{dP_\ell} = -\frac{1}{P_\ell^2}\frac{(1+x)^2(1+xM_3)}{x\{1+x(1+M_1)M_3\}}, \tag{34}$$

$$\frac{dR_1}{dM_3} = -\frac{M_1(1+x)^2}{P_\ell\{1+xM_3(1+M_1)\}^2}. \tag{35}$$

This shows that, for a stationary convection, the medium permeability and non-buoyancy magnetization are found to have destabilizing effect on the system.

The critical Rayleigh number for the onset of instability is determined by the condition $dR_1/dx = 0$. When $M_1 = 0$, then from equation (32), we have

$$a_c^2 = \pi^2 \text{ with } R_c = \frac{1}{P_\ell}4\pi^2.$$

For M_1 sufficiently large, we obtain the results for the magnetic mechanism operating in porous medium

$$N = R_1 M_1 = \frac{(1+x)^2(1+xM_3)}{P_\ell x^2 M_3}, \tag{36}$$

where N is the magnetic thermal Rayleigh number.

The critical magnetic Rayleigh number for the onset of instability is determined by the condition $dN/dx = 0$; we get

$$N_c = \frac{(1+x_c)^2(1+x_c M_3)}{P_\ell x_c^2 M_3}, \quad \text{where} \quad x_c = \frac{M_3 + \sqrt{M_3^2 + 8M_3}}{2M_3}. \tag{37}$$

The critical wave-number and critical magnetic number N_c depend on the non-buoyancy magnetization parameter M_3 and medium permeability parameter P_ℓ, taking the values

$$a_c^2 = 2\pi^2\,,\; N_c = \frac{1}{P_\ell}\frac{27}{4}\pi^2 \quad \text{for} \quad M_3 = 1$$

and

$$a_c^2 = \pi^2\,,\; N_c = \frac{1}{P_\ell}4\pi^2 \quad \text{for} \quad M_3 \to \infty\,,$$

and intermediate values for intermediate M_3.

The dispersion relation (32) is analyzed numerically. In figure 2, R_1 is plotted against wave number x for $M_1 = 1000$, $M_3 = 1$; $P_\ell = 0.001$, 0.002, 0.003 and 0.004. In figure 3, R_1 is plotted against wave number x for $M_1 = 1000$, $P_\ell = 0.001$; $M_3 = 1$, 3, 5, 7. It is clear that the medium permeability and non-buoyancy magnetization have destabilizing effect as the Rayleigh number decreases with the increase in medium permeability parameter and non-buoyancy magnetization parameter. In figure 4, the critical magnetic thermal Rayleigh number N_c is plotted against the non-buoyancy magnetization M_3 for $P_\ell = 0.001$, 0.002, 0.003 and 0.004. This shows that as the non-buoyancy magnetization parameter M_3 and the Darcy number P_ℓ increase, the critical magnetic Rayleigh number N_c decreases. Therefore, lower values of N_c are needed for the onset of convection with an increase in M_3 and P_ℓ, hence justifying the destabilizing effect of non-buoyancy magnetization M_3 and medium permeability P_ℓ (table).

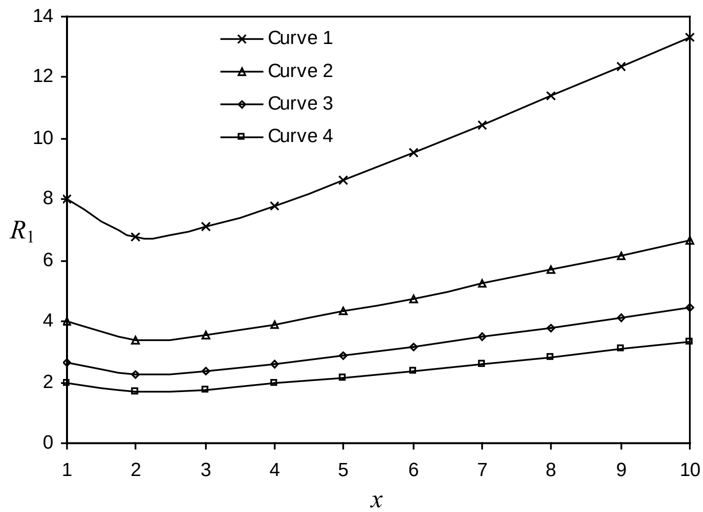

Fig. 2. The variation of the Rayleigh number (R_1) with the wave number (x) for $M_1 = 1000$, $M_3 = 1$; $P_\ell = 0.001$ for curve 1, $P_\ell = 0.002$ for curve 2, $P_\ell = 0.003$ for curve 3 and $P_\ell = 0.004$ for curve 4

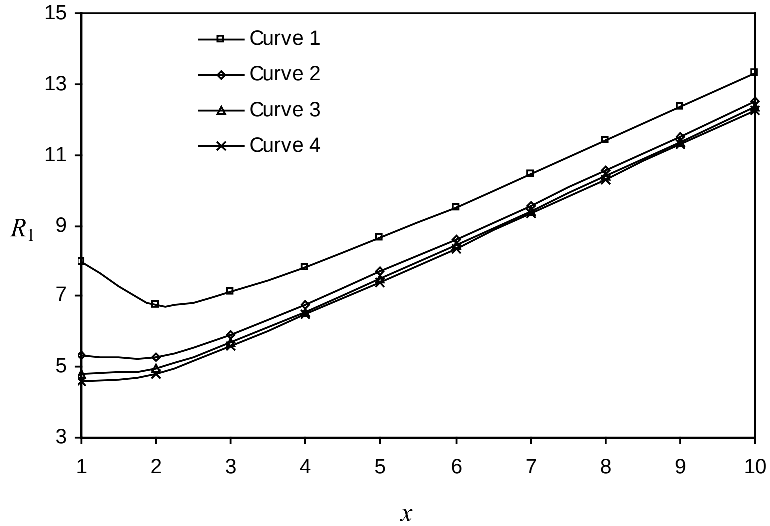

Fig. 3. The variation of the Rayleigh number (R_1) with the wave number (x) for $M_1 = 1000$, $P_\ell = 0.001$; $M_3 = 1$ for curve 1, $M_3 = 3$ for curve 2, $M_3 = 5$ for curve 3 and $M_3 = 7$ for curve 4

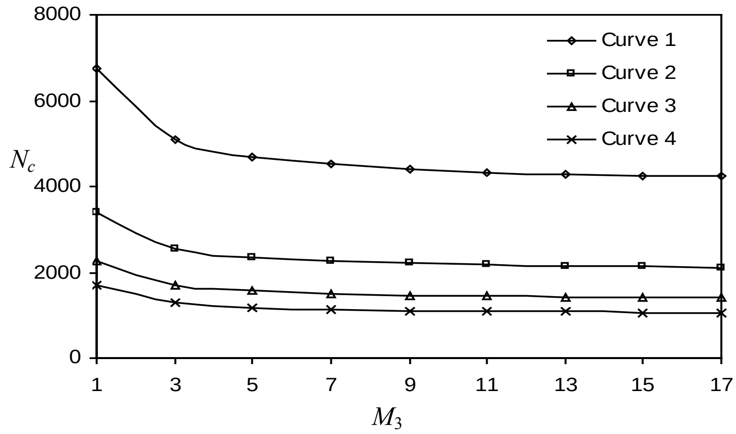

Fig. 4. The variation of the critical magnetic Rayleigh number (N_c) with non-buoyancy magnetization (M_3) for $P_\ell = 0.001$ for curve 1, $P_\ell = 0.002$ for curve 2, $P_\ell = 0.003$ for curve 3 and $P_\ell = 0.004$ for curve 4

Suggestion from FINLAYSON [8] has also been taken for a variation of these parametric values. In the present analysis, the range of values pertaining to ferric oxide, kerosene and other organic carriers are chosen. With the same ferric oxide, the different carriers like alcohol, hydrocarbon, ester, halocarbon, silicon could be chosen. Depending on this, the parametric values of ferromagnetic fluid are found to vary within these limits. For such fluids, the typical values of M_2 are $+10^{-6}$ and so is assumed to have a negligible value and hence it is taken to be zero (FINLAYSON [8]). The non-buoyancy magnetization parameter M_3 measures the departure of linearity in the magnetic equation of state and the values from one $(M_0 = \chi H_0)$ to higher values are possible for the usual equation of state. Thus M_3 is varied from 1 to 25.

Table

Critical magnetic thermal Rayleigh numbers and wave numbers of the unstable modes at marginal stability for the onset of stationary convection

M_3	x_c	$(P_\ell = 0.001)$ N_c	$(P_\ell = 0.002)$ N_c	$(P_\ell = 0.003)$ N_c	$(P_\ell = 0.004)$ N_c
1	2	6750	3375	2250	1687.5
3	1.457427	5091.258	2545.629	1697.086	1272.814
5	1.306226	4695.234	2347.617	1565.078	1173.808
7	1.231925	4512.576	2256.288	1504.192	1128.144
9	1.187184	4406.644	2203.322	1468.881	1101.661
11	1.157129	4337.271	2168.635	1445.757	1084.318
13	1.135489	4288.24	2144.12	1429.413	1072.06
15	1.119139	4251.717	2125.858	1417.239	1062.929
17	1.106339	4223.443	2111.721	1407.814	1055.861

5.2. PRINCIPLE OF EXCHANGE OF STABILITIES

Here we examine the possibility of the effect of oscillatory modes, if any, on stability problem due to the presence of magnetization parameters and medium permeability. Equating the imaginary parts of equation (31), we obtain

$$\sigma_i\left[\frac{1}{\varepsilon}(1+x)^2(1+xM_3)+\frac{1}{P_\ell}(1+x)\{(P_r'-\varepsilon P_r M_2)+xP_r'M_3\}\right]=0. \tag{38}$$

Here the quantity inside the brackets has a positive definite form because the typical values of M_2 are $+10^{-6}$ (FINLAYSON, [8]). Hence

$$\sigma_i = 0. \tag{39}$$

This shows that whenever $\sigma_r = 0$ implies that $\sigma_i = 0$, then the stationary (cellular) pattern of flow prevails upon the onset of instability. In other words, the principle of exchange of stabilities is valid for the ferromagnetic fluid heated from below saturating a porous medium.

6. CONCLUSIONS

In this paper, we studied the thermal instability of a ferromagnetic fluid saturating a porous medium in the presence of uniform vertical magnetic field. We have investigated the effects of medium permeability and non-buoyancy magnetization on the onset of convection. The principal conclusions from the analysis of this paper are as follows:

(i) For stationary convection, the medium permeability and non-buoyancy magnetization are found to have a destabilizing effect on the system which has been demonstrated both analytically and numerically. It is clear from figures 2 and 3 that the medium permeability and non-buoyancy magnetization have a destabilizing effect as the Rayleigh number decreases with the increase in the Darcy number and non-buoyancy magnetization parameter.

(ii) The critical wave numbers and critical magnetic thermal Rayleigh numbers for the onset of instability are also determined numerically for sufficiently large values of buoyancy magnetization M_1 and the results are depicted graphically. For sufficiently large values of buoyancy magnetization M_1, figure 4 and table show that as non-buoyancy magnetization parameter M_3 and Darcy number P_ℓ increase, the critical magnetic Rayleigh number N_c decreases. Therefore, lower values of N_c are needed for the onset of convection with an increase in M_3 and P_ℓ, hence justifying the destabilizing effect of non-buoyancy magnetization M_3 and medium permeability P_ℓ. In order to investigate our results, we must review the results and their physical explanations. It is well known that the when fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium, then the medium permeability has a destabilizing effect. As medium permeability increases, the void space increases and as a result of this, the flow quantities perpendicular to the planes will clearly be increased. Thus an increase in heat transfer is responsible for the early onset of convection. Thus increasing P_ℓ leads to a decrease in N_c. The increase in non-buoyancy magnetization as well as in medium permeability is found to cause large destabilization, because medium permeability, magnetic and thermal mechanisms favour destabilization.

(iii) The principle of exchange of stabilities is found to hold true for the ferromagnetic fluid saturating a porous medium heated from below.

Thus from the above analysis, we conclude that the medium permeability has a profound influence on the onset of convection. It is hoped that the present work will serve as a vehicle for understanding more complex problems investigated in the present paper.

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