Studia Geotechnica et Mechanica, Vol. XXVII, No. 3–4, 2005

USE OF INTERPOLATION METHODS FOR GEOTECHNICAL PROFILING

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1. INTRODUCTION

Natural soils were formed due to geological processes taking place for thousands and millions of years. Effects of these processes often overlapped, eventually creating forms of frequently differentiated features and clear spatial variability. While analyzing subsoil for design purposes a geotechnician or engineer geologist aims, according to designer expectations, at obtaining a possibly generalized picture of the subsoil structure. This analysis is carried out based on the data from, most often, only point measurements taken in test profiles. Designing a structure foundations, however, requires spatial assessment of the distribution of geotechnical parameters. During construction of a subsoil model corresponding to both reality and design requirements, the knowledge and experience of the researcher are of a great importance. At present, their work can be supported by such modern methods of analysing the distribution of subsoil features as various interpolation techniques. Application of these techniques allows drawing maps of both measured parameters and any other subsoil feature. The data mapping methods may be applied to one feature only (such as kriging) or allow taking into account several parameters at the same time (e.g. using the method of canonical correlations).

Cone penetration test (CPTU) and flat dilatometer tests (DMT) taking a dominating position among the in situ methods give wide possibilities of construction subsoil recognition. Lower costs of in situ tests compared to the cost of obtaining high-quality samples from test drills and carrying out laboratory tests allow densification of the test point grid. At the same time high frequency of registering test parameters allows almost continuous observation of changes taking place along the profile studied. In this paper, an example of detecting spatial differentiation of subsoil structure from the CPTU test results is presented.

2. TEST SITE

In order to present and to analyze the possibilities of applying modern interpolation techniques, the results of geotechnical studies carried out within the project of corn silos in Borek Strzeliński, Poland, were used. In the subsoil of the area studied, cohesive sediments predominate. They are formed as moraine loams of the Odra glaciation and considered to be overconsolidated soils. Among the moraine sediments there are present, typical of this facies, sandy interbeddings and layers of fluvioglacial coarse and medium sands.

On the test site 10 CPTU tests were made as well as one bore hole to take samples for laboratory tests and to calibrate in situ tests. Two CPTU parameters were used in the analysis – normalized cone resistance Q_t and friction ratio R_f. The former is connected with strength characteristics of the soil, while the latter is related to the kind of the soil tested [4].

3. SELECTED INTERPOLATION TECHNIQUES AND THEIR APPLICATION

Presenting theoretical bases of selected methods it should be pointed out that the techniques of data mapping not only have been known and developed for many years, but also applied to many fields of science and industry (e.g. the so-called geostatics, mining industry) [1]. The methods discussed below can be used not only in geotechnics, provided that mathematical criteria are strictly fulfilled. Accepting initially such a generalization we can assume that we are interested in the variable Z whose value we want to estimate in a certain area D. This variable can be a test parameter or a subsoil feature. From in situ (or laboratory) tests we have observations of the variable Z in the area D in n specific locations (test points):

$$Z(s_1), ..., Z(s_n), \quad s_k = (x_k, y_k) \in D, \quad k = 1, ..., n. \tag{1}$$

The number of test points can range from several to several hundred, but we assume that it would never allow determination of the value of the variable Z in the area studied. The techniques discussed in this paper enable design of a model of the value distribution for this variable. Thus, each time (independently of the method used) we obtain a map of the variable Z estimated values on an assumed plane X, Y of the area D.

Among the simplest interpolation methods are the techniques of local smoothing. One of them is the ABOS method (Approximation Based on Smoothing) [2]. The smoothed value of the surface at any bilinear point $s = (x, y)$ within the gird can be evaluated from the equation of bilinear polynomial

$$Z(s) = axy + bx + cy + d, \quad s = (x, y) \in D, \tag{2}$$

which is defined by corner points of square containing point s.

Accepting, for example, a static sounding parameter Q_t as the variable Z, we obtain a map of this parameter in the area assumed, in our case at a given depth (figure 1). The value of the parameter estimated can be read out directly from the map. The ABOS method is a typical analytic technique in which a statistical model of variability of a feature is not determined, but more and more smooth surface is obtained in the subsequent steps.

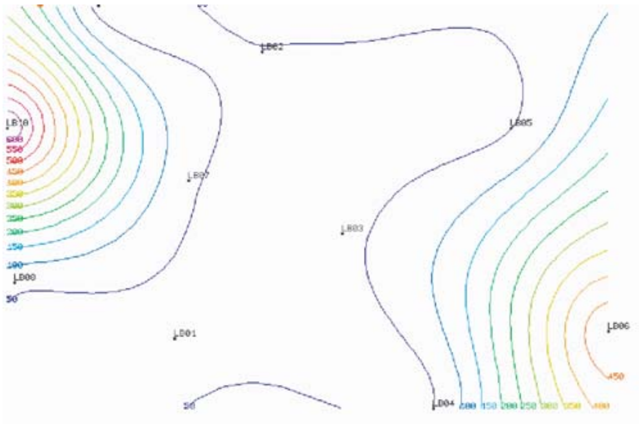

Fig. 1. The map of the parameter Q_t derived by ABOS method
(at the depth of 2 m)

A more advanced method of estimating the value of our variable is kriging [3]. In this method, to estimate an unknown value of the variable Z in a point s of the area D, we use not only the specified values of this variable in the test points s_1, ..., s_n, but also a semi-variogram describing a spatial variation. The kriging estimator has the following form:

$$Z(s) = \sum_{k=1}^{n} \lambda_k Z(s_k) \,. \tag{3}$$

We assume the following model of spatial variation:

$$Z(s) = \mu(s) + \delta(s), \quad s \in D, \tag{4}$$

where $\mu(s)$ is a trend, and $\delta(s)$ – a noise.

Using the method of ordinary kriging we accept the following assumption for the trend

$$\mu(s) = \mu. \tag{5}$$

In the case of the noise, it is assumed that it is a stationary random process for which

$$\mathrm{E}\,(\delta(s)) = 0, \tag{6}$$

$$\mathrm{Var}\,(\delta(s+h) - \delta(s)) = 2\gamma(h). \tag{7}$$

We also assume that this process is isotropic, i.e.:

$$2\gamma(h) = 2\gamma_0(\|h\|). \tag{8}$$

As can be seen, in this method the calculations are much more complicated than in the first one, although the results do not differ significantly (figure 2).

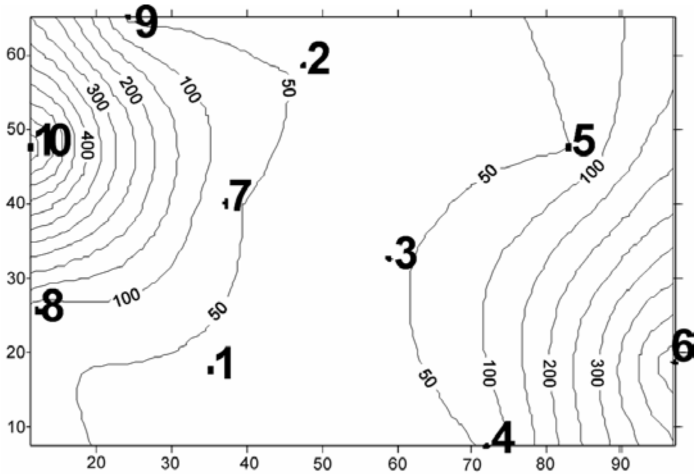

Fig. 2. The map of the parameter Q_t derived by kriging method (at the depth of 2 m)

In the case of this method, the surface obtained greatly depends on the assumed statistical model of the feature variability. In the example discussed, a linear model was used for semivariogram. Its form is as follows:

$$2\gamma_0(\|h\|) = c_0 + c\|h\|, \tag{9}$$

where c_0 and c are the parameters.

The possibility of applying other models was limited due to an insufficient number of measurement points.

The coefficients $\lambda_1, \ldots, \lambda_n$ of the kriging estimator are obtained by minimizing the expected square error of the estimation

$$\mathrm{E}\left[Z(s) - \sum_{k=1}^{n} \lambda_k Z(s_k)\right]^2, \quad \sum_{k=1}^{n} \lambda_k = 1\,. \tag{10}$$

This technique leads to the solution of the system of equations

$$\sum_{j=1}^{n} \lambda_j \gamma(s_i - s_j) + m = \gamma(s - s_i), \quad i = 1, ..., n, \tag{11}$$

$$\sum_{i=1}^{n} \lambda_i = 1, \tag{12}$$

where m is Lagrange's multiplier. Thus the estimator obtained is unbiased and has minimal variance.

Another group of methods includes the technique based on canonical correlations. Their application enables analysis of several variables simultaneously. This allows, for example, considering a simultaneous effect of different test parameters on the value of the soil properties determined.

In this method, we assume that in the area D studied, p-variables $Z_1, \ldots, Z_p$ are analyzed. As in the previous case we have observations of these variables for n locations $s_1, ..., s_n$ from the area D. A point $s = (x, y)$ is determined by geographical (spatial) variables. Hence, we have two groups of variables: the first describing the features of the area D: $Z_1, ..., Z_p$, and the second describing geographical position $Y_1, ..., Y_q$ which, depending on the assumed degree of the model, is as follows:

$Y_1 = X$, $Y_2 = Y$ for the linear model,

$Y_1 = X$, $Y_2 = Y$, $Y_3 = XY$, $Y_4 = X^2$, $Y_5 = Y^2$ for the quadratic model, etc.

New canonical variables are linear combinations of the initial variables:

$$U = \mathbf{a}\mathbf{Z}', \quad V = \mathbf{b}\mathbf{Y}', \tag{13}$$

where

$$\mathbf{a} = (a_1, ..., a_p), \quad \mathbf{Z} = (Z_1, ..., Z_p),$$

$$\mathbf{b} = (b_1, ..., b_q), \quad \mathbf{Y} = (Y_1, ..., Y_q),$$

determined in such a way that

$$\rho(U, V) = \max.$$

The vectors **a** and **b** are obtained from the solution of the system of equations:

$$\begin{aligned} (S_{12}S_{22}^{-1}S_{12}' - \rho^2 S_{11})\mathbf{a} = 0, \\ (S_{12}S_{12}^{-1}S_{12}' - \rho^2 S_{22})\mathbf{b} = 0, \end{aligned} \tag{14}$$

where ρ^2 is a maximal root of the equation:

$$| S_{12}'S_{11}^{-1}S_{12} - \rho^2 S_{22} | = 0. \tag{15}$$

The matrices S_{11}, S_{12}, S_{21}, S_{22} are the elements of the covariance matrix of the $p + q$ random variables $Z_1, \ldots, Z_p, Y_1, \ldots, Y_q$ in the following form:

$$S_{ZY} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}. \tag{16}$$

Estimator of canonical variable U in the area D is obtained using a linear regression between the variables U and V.

Using this method we can draw a map taking into account simultaneous variability of two CPTU parameters Q_t and R_f (figure 3).

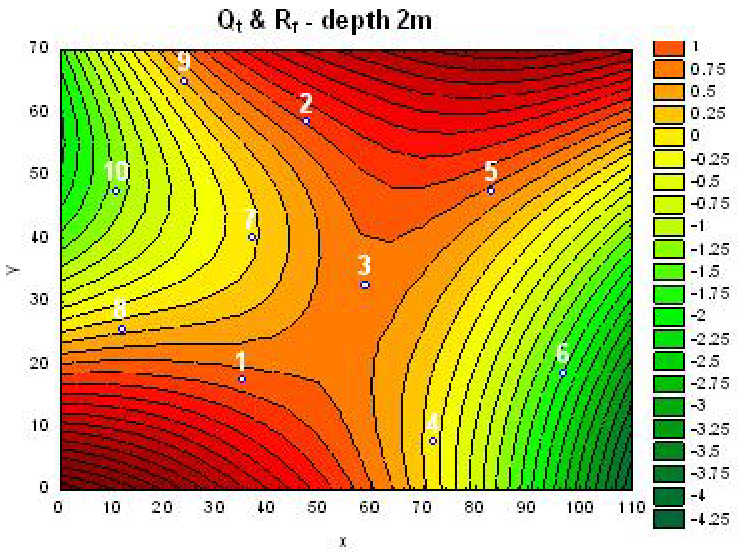

Fig. 3. Map of combined variable R_f and Q_t obtained by means of canonical correlation method (at the depth of 2 m)

The calculation techniques under discussion allow us to draw the subsoil maps when a given depth or strain level σ_{v0} is known. In order to obtain a 3D graphical representation, such maps are plotted at subsequent levels and set one over another giving a certain idea of spatial distribution of the parameter studied (figure 4).

However, in some cases such a picture may be unclear or insufficiently accurate. By means of digital processing of the maps there is a possibility of creating the cross-sections of given levels in any place of the area under study (not necessarily including test points) (figure 5). An appropriate setting of such cross-sections for many levels enables creation of variability of a given parameter on practically any cross-section planes (figure 6).

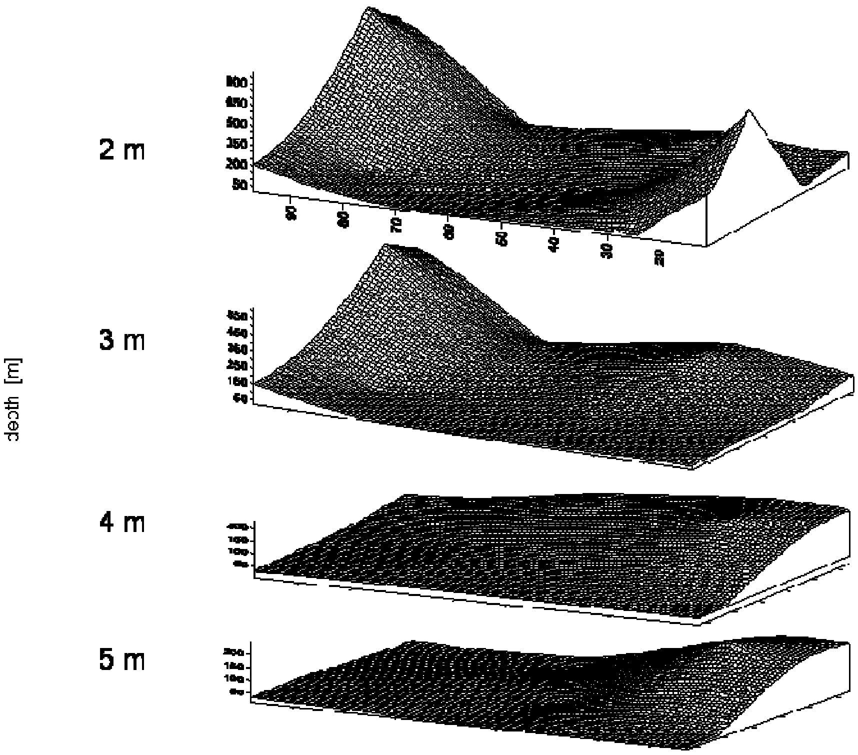

Fig. 4. A 3D picture of spatial variability of the parameter Q_t derived by kriging method

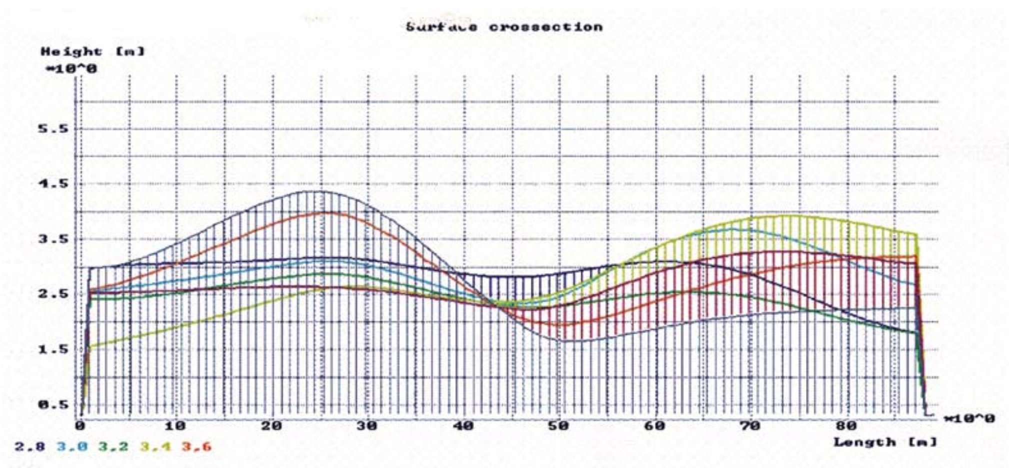

Fig. 5. The distribution Q_t at different depths along the cross-section under consideration derived by ABOS method

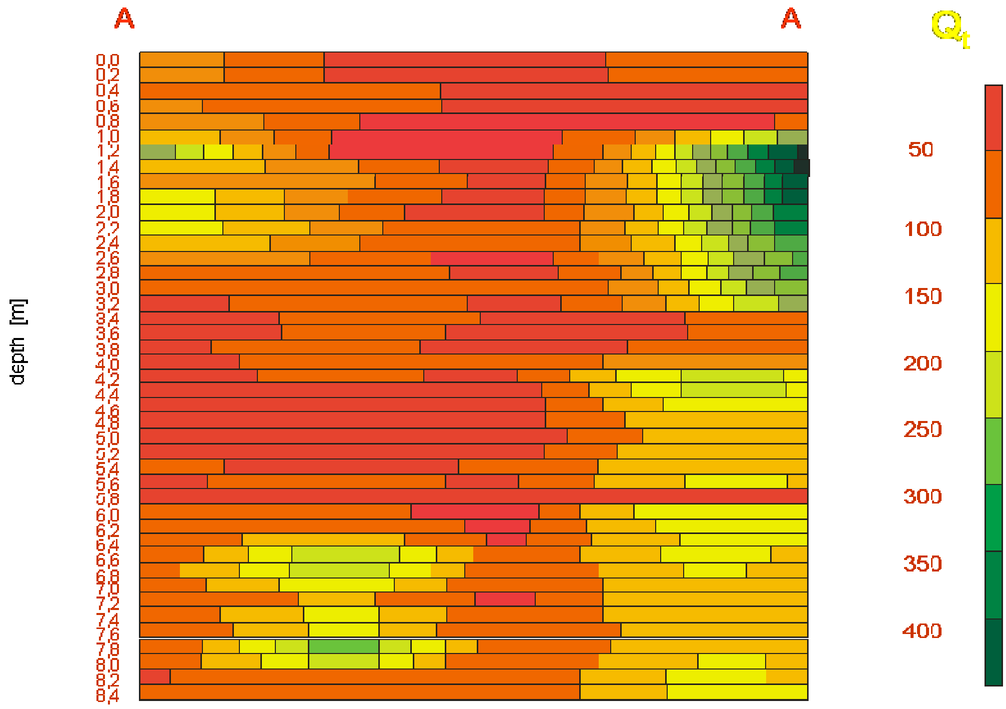

Fig. 6. Changes in Q_t on the cross-section surface derived by ABOS method

4. CONCLUSIONS

Contemporary interpolation techniques used in science can find a broad application in geotechnics. The maps of geotechnical parameters give valuable information about subsoil structure which can be easily used by a designer. Among the advantages of this method is separation of the so-called weak subsoil layers which determines the method of founding an object.

At a relatively small number of measurement points (as in the example presented) no differences are observed in the effects obtained using various interpolation techniques, or at least these differences are not significant for a designer. Increasing the number of test points enables us to apply the functions characterizing subsoil variability and being more complex than linear model. In such a case, the picture of subsoil obtained from kriging and canonical correlations will undergo changes.

Application of digital interpolation techniques in order to create geological or geotechnical cross-sections is a new approach. It allows taking into account the effect of the parameter values measured in adjacent subsoil points on an the assumed parameter value in a given point of subsoil space. Hence, it is a step forward in obtaining fully three-dimensional model of the subsoil structure. Since the proposed technique of creating the model of subsoil structure allows determination of the value of a trait studied in any point of a three-dimensional space, it enables obtaining geotechnical cross-sections in any plane, not only in perpendicular ones. This makes it possible to draw maps of soil parameter distribution along, for example, potentially slide surfaces.

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