Studia Geotechnica et Mechanica, Vol. XXVI, No. 3–4, 2004

COMPUTATIONAL MODEL FOR JET-GROUTING PILE – SOIL INTERACTION

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Streszczenie: Zaproponowano model numeryczny pala wykonanego techniką wysokociśnieniowej iniekcji strumieniowej i współdziałającego z nim masywu gruntowego. Model budowano w czterech etapach: tworzenie koncepcji i matematyczne formułowanie, kalibrowanie, weryfikacja i analiza jego wrażliwości (BZÓWKA [7], [8], [9]). Teza artykułu brzmi: Model numeryczny pala strumieniowo-iniekcyjnego i procedury jego kalibrowania mają umożliwić realistyczne przewidywanie osiadania układu pal strumieniowo-iniekcyjny–grunt i wytężeń jego tworzywa w szerokim przedziale obciążenia.

Koncepcyjnie model w sposób zasadniczy odbiega od bardzo nielicznych istniejących propozycji obliczeniowych, które są adaptacjami półempirycznych wzorów używanych do obliczania nośności
i osiadania pali żelbetowych. Modelowano fizyczną rzeczywistość, rozważając współśrodkowy układ trzech oddziałujących na siebie ciał o kształcie cylindrycznym i właściwościach sprężysto-plastycznych. Do opisu mechanicznego zachowania się każdej ze stref przyjęto najprostszy model konstytutywny przyrostowej teorii plastyczności – ośrodek sprężysto-idealnie plastyczny zdefiniowany za pomocą warunku stanu granicznego Coulomba–Mohra i stowarzyszonego z nim prawa płynięcia. Metodyka kalibrowania parametrów modelu podporządkowana została dwóm generalnym ideom:

- spójnej bazy własnych danych eksperymentalnych w procesach kalibrowania i weryfikacji modelu;
- oddzielnego szacowania parametrów dla każdej strefy.

Weryfikacja odgrywała szczególną rolę w budowie modelu obliczeniowego, dostarczyła bowiem odpowiedzi na pytanie, czy i w jakim stopniu jest on efektywny w aspekcie możliwości symulacyjnych i stosowalności w praktyce. Dowód na słuszność postawionej w pracy tezy ma postać porównania teoretycznej i eksperymentalnej charakterystyki obciążenie – osiadanie doświadczalnego pala strumieniowo-iniekcyjnego.

Abstract: Computational model for jet-grouting pile–soil system is presented. Construction of the model proceeded in four phases: conception creating and mathematical formulating, model's parameters calibrating, verification and sensitivity analysis (BZÓWKA [7], [8], [9]). Thesis of the paper is as follows: Numerical model for jet-grouting pile and procedures of its calibration have to allow realistic prediction of jet-grouting pile–soil system settlement and efforts prediction of pile's materials at a wide range of loading.

Computational model is completely different than other sparsely existing proposals that are adaptations of semi-empirical formulas used for calculating loads and settlement of reinforced concrete piles. In the paper, physical reality was modelled. A numerical model contains three concentric, cylindrical, influencing zones which have got elastic-plastic properties. For mechanical behaviour of each zone the Coulomb–Mohr model was used. Methodology of parameter's calibration was conformed to two main ideas:

- coherent base of experimental data in the calibration and verification processes;
- separate estimation of parameters for each zone.

Verification was really important for the construction of computational model. It gave the answer to the question whether or to what extent the model is effective in terms of the possibility of its simulation and applicability in practice. To prove the thesis of the paper theoretical and experimental load–settlement curves representing empirical jet-grouting pile were compared.

Резюме: В статье предлагается нумерическая модель сваи выполненной техникой струйной инъекции высокого давления и взаимодействующим с ней массивом грунта. Создание модели проходило в четыре этапа: создание концепции и математической формулировки, калибровка, проверка и анализ чувствительности модели (BZÓWKA [7], [8], [9]).

Тезис статьи: «Нумерическая модель струйно-инъекционной сваи и процедуры ее калибровки должны создать возможность реально предусмотреть оседание системы: «струйно-инъекционной сваи – грунт и натяжений материала в широком диапазоне нагрузок».

В концепционном значении модель существенным образом отличается от немногих существующих расчётных предложений, которые являются полуэмпирическими адаптациями формул используемых для расчетов грузоподъёмности и оседания железобетонных свай. Моделировалась физическая действительность, рассматривая концентрическую систему трёх действующих друг на друга сил цилиндрической формы и упруго-пластичными свойствами. С целью описания механического поведения каждой из зон была принята простейшая модель основной теории приращения пластичности – центр упруго-идеально пластичный, определяемый с помощью условия предельного состояния Coulomba-Mohra и взаимодействующего с ним закона текучести. Метод калибровки параметров модели был подчинён двум генеральным идеям:

- Сплачению базиса собственных экспериментальных данных в процессах калибровки и проверки модели.
- Отдельной оценки параметров для каждой зоны.

Проверка играла особенную роль в строении расчётной модели, дала ответ на вопрос есть ли и в какой степени модель эффективна в аспекте возможности имитирования и применения на практике. Доказательством правильно поставленного в работе тезиса, является сравнение теоретической и экспериментальной характеристики «нагрузка–оседание» опытной струйно-инъекционной сваи.

1. INTRODUCTION

Jet grouting is a method, which uses the effect of shearing and disintegrating the soil by exposing it to operation of cement grout jet. The jet flows out of a nozzle at the speed of 100 m/s and the pressure of 15–70 MPa. Particles of soil surrounded by the grout fill up the space within erosion reach of jet and the excess of particles flows out onto the surface. The jet-grouting method was invented and developed in Japan in the early 1970s and it was used for the first time by Kajima Corporation, the company from Tokyo. The method spread across Europe only in the 1980s. For the first time the method was applied by Swedish company called Geo Projektering in co-operation with Chemical Grouting Company. They were to provide protection to the building foundations in the old city center in Stockholm (SZYMANKIEWICZ [32]; KŁOSIŃSKI [21]; BORYS et al. [2]). Jet-grouting method was then also used in Germany, Holland, the USA, Italy, Great Britain and in the former Soviet Union. Analogous method was then developed in the Research Institute of Foundation Engineering and Underground

Structures (NIIOSP) in Moscow.

Typical course of jet-grouting includes the following stages (figure 1):

- drilling a hole of the diameter of about 100 mm up to the anticipated depth,
- cutting the soil with thin stream of water or cement grout at the pressure ranging from 15 to 70 MPa; in order to increase the jet energy it can be forced with a shield of compressed air; the water together with the excavated material flows out through the hole onto the surface, or in case of walls forming – also through the adjoining "relieving" hole,
- simultaneous forming the element out of cement grout.

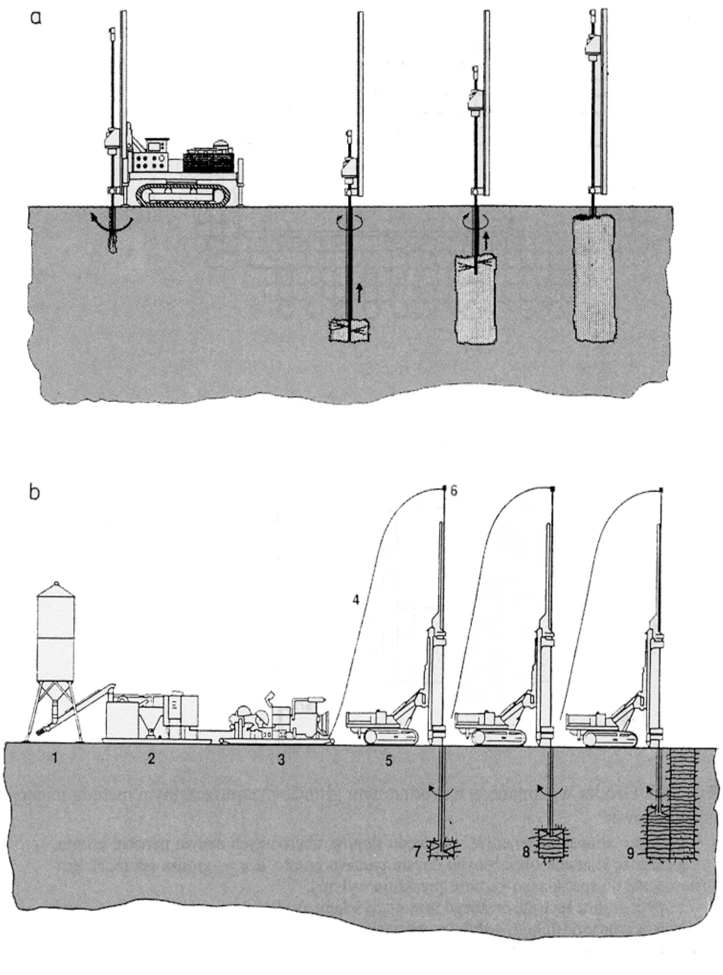

Fig. 1. Stages of conducting jet-grouting method (a) as well as set of the equipment used (b):
1 – cement silo, 2 – cement-inject plant, 3 – high-pressure pump, 4 – high-pressure conduit,
5 – rotary drilling rig, 6 – casing head, 7 – beginning of the jet injection after having driven a drilling rod until the designed depth, 8 – jet petrification of the first pile, 9 – next pile forming

(after JAROMINIAK [20])

Mono-, double- and triple-jet grouting methods are distinguished. Mixture of cement and subsoil, called cement-soil, which is formed as a result of high-pressure jet injection, after it has been hardened, gains considerable compression strength ranging from 2 to 25 MPa, which depends on the kind of soil, accompanied by a decreasing value of water-permeability coefficient modulus by a few orders of magnitude (ŻMUDZIŃSKI and MOTAK [38]). While applying high-pressure jet-injection method, what is obtained, contrary to other types of injection, is uniform distribution of binder or sealing material in the soil, irrespective of its original graining, strength and cracks that occur. This results in a unification of soil characteristic as well as in an increase of soil strength and tightness (PETYNIAK and KŁOSIŃSKI [26]).

Technology of high-pressure jet injection has several advantages. Lumps of cement-soil can be shaped in any geometrical form, at required place in subsoil and almost in any soil. Usually jet-grouting piles are formed and they are being considered in the article presented.

Other forms and engineering structures are as follows: sealing screens, lamel walls, cell walls, tight palisades, etc. Advantages of the jet-grouting method are particularly useful when deep excavation has to be dug very close to the existing structure, when there is not enough space for heavy building equipment or when other methods of improving geotechnical conditions are unacceptable due to vibrations that may affect badly either people or nearby structures. The jet-grouting method is recommended especially for strengthening earthen foundation under the structures that have been settling too much to make vertical and horizontal water-proof cut-off walls (in excavations, in flood banks), for outer casing of tunnels as well as for making designs of foundations for new buildings.

2. CLASSICAL METHODS OF DIMENSIONING

2.1. CURRENT STATE

Many scientists point out that the jet-grouting method is not commonly used due to lack of method of pile load capacity assessment at the design stage. Pile standard PN-83/B-02482 [29], which is valid, does not includes regulations concerning designing piles which are to be made with this method; until 1995 neither domestic nor foreign publications were known.

In order to establish preliminary suggestions for computational assessment of jet-grouting pile load capacity, Polish scientists, ŻMUDZIŃSKI and MOTAK [37] as well as GWIZDAŁA and MOTAK [18], referred to hitherto results of loading tests and to their own analysis, and they also made use of the analogy between this type of piles and injection micropiles.

In the project of European standard (proposed EN 12716:1997 [14]) on executing special geotechnical works – jet-grouting, very little was said about design requirements.

Design assumption, according to the chapter 2 ENV 1997-1 "Eurocod 7 – part 1", should be verified and modified when necessary in accordance with data gained during the execution phase. It is necessary to check anticipated stress in the injection elements, including possible influence of soil changeability on the strength of such elements. Eurocod 7 requires also checking general stability of injection elements and structures used to reinforce the foundation or as retaining walls, including partial factors of safety in compliance with Eurocod 7.

Generally, pile foundation, without distinction between the types of piles, was classified as geotechnical category 2. For every design situation it is necessary to check whether limiting states will not be exceeded. Limiting states may occur in the subsoil or in the structure and subsoil together. They can be checked for computational models (analytical, semi-empirical, numerical) on the grounds of regular proceedings (proving proceedings based on checking an appropriate selection and control of the materials used) or on the grounds of trial loads and model testing. It is also possible to check limiting states by observation methods where the project is corrected during the building works.

In the ways proposed to evaluate load capacity of new piles presented by Polish authors recently, basic formulae of Polish standard PN-83/B-02482 [29] have been used:

$$Q_r \le m \cdot N\,, \tag{1}$$

where:

$$N = N_t = S_p \cdot q^{(r)} \cdot A_p + \sum S_{si} \cdot t_i^{(r)} \cdot A_{si} \tag{2}$$

for the compressing piles,

$$N = N^w = \sum S_{si}^w \cdot t_i^{(r)} \cdot A_{si} \tag{3}$$

for the uplifting piles.

In expressions (1)–(3)

Q_r – computational pile load, compressing or uplifting,

m – correcting coefficient, depending on the number of piles below the foundation,

N_t, N^w – computational load capacities of the compressing and uplifting piles,

S_p, S_{si}, S_{si}^w – technological coefficients, depending on the type of pile and soil,

$q^{(r)}$, $t_i^{(r)}$ – unit computational resistance of soil under the base or along the pile shaft,

A_p, A_{si} – surfaces of the base and shaft of the pile.

ŻMUDZIŃSKI and MOTAK [37] on the grounds of the results of a large number of French tests and experiments recommend the following formulae:

for compression:

$$N_t = 1.1 \cdot \sum t_i^{(r)} \cdot A_{si} \,, \tag{4}$$

for uplifting:

$$N^w = \sum t_i^{(r)} \cdot A_{si} \,, \tag{5}$$

where $t_i^{(r)} = \gamma_m \cdot t_i^n$ is a computational value of unit shaft resistance. It is assumed that $m = 0.7$ in the case of compression piles, while in the case of uplifting piles $m = 0.65$. Characteristic values of unit frictional resistance t^n along the shaft of pile for different kinds of non-cohesive and cohesive soils should be taken according to tables 1 i 2 (ŻMUDZIŃSKI and MOTAK [37]). The values are 1.3–2.6 times (in average approx. 1.9 times) as big as in the pile standard (PN-83/B-02482 [29]) for non-injection piles.

Different proposition for estimation of load capacity of compressing jet-grouting piles was presented by GWIZDAŁA and MOTAK [18] after having tested five piles. They suggest using the following formula for compressing piles:

$$N_t = q^{(r)} \cdot A_p + \sum t_i^{(r)} \cdot A_{si} \,, \tag{6}$$

where:

$q^{(r)} = \gamma_m \cdot q^n$ – computational unit base resistance, kPa,

q^n – value of unit base resistance considering the pile base hollow, kPa ,

$\gamma_m \le 0.9$ – material coefficient, $\gamma_m = \gamma_m(I_L)$ according to Polish standard PN-81/B-03020 [28], as well as assuming values of correction coefficient $m = 0.7$ and technological coefficients $S_p = S_{si} = 1.0$.

The authors recommend taking in formula (6) the value q^n according to the pile standard (PN-83/B-02482) as for non-injection piles (tables 3 & 4), and value t^n as for jet-grouting piles according to ŻMUDZIŃSKI and MOTAK (tables 1 & 2), (ZADROGA [33]).

Table 1

Characteristic values of unit friction resistance t^n [kPa] along the shaft of the jet-grouting piles in non-cohesive soil (dependent on the density index I_D)

Soil type	t^n [kPa] $I_D = 0.20$	t^n [kPa] $I_D = 0.33$	t^n [kPa] $I_D = 0.50$	t^n [kPa] $I_D = 0.67$	t^n [kPa] $I_D = 0.80$	t^n [kPa] $I_D = 0.90$
Gravel, sandy gravel	60	83	115	140	163	180
Coarse and medium sand	50	72	105	130	153	160
Fine and silty sand	45	60	82	100	114	125

Table 2

Characteristic values of unit friction resistance t^n [kPa] along the shaft of the jet-grouting piles in cohesive soil (dependent on the liquidity index I_D)

Soil type	t^n [kPa] $I_L < 0$	t^n [kPa] $I_L = 0$	t^n [kPa] $I_L = 0.25$	t^n [kPa] $I_L = 0.50$	t^n [kPa] $I_L = 0.65$	t^n [kPa] $I_L = 0.75$
Loam, loamy sand	135	110	80	50	30	18
Firm loam, clay	125	100	70	40	25	14
Silt, sandy silt	110	85	75	35	20	9

The research by GWIZDAŁA and MOTAK [18] is different from the others also because it is not restricted to anticipation of load capacity. It also includes the procedure of predicting settlements with application of the so-called functions of transformation $q - z$ and $t - z$. The functions present empirical description of pile settlement dependence on unit resistance below the base and along the shaft, respectively. They are selected on the grounds of experimental database collected from model tests in different scales and from monitoring conducted during trial loading of the piles. In the case of different types of reinforced concrete piles, the database can be said to be quite large. However, it is not so, as far as jet-grouting piles are concerned. Hence there is more uncertainty about an appropriate selection of functions of transformation. As empirical ones, the functions do not provide solution to a question of mechanics, but they are the result of optimum fitting to the results of specific experiments.

Table 3

Values of unit limiting soil resistance under the pile base q^n [kPa] for non-cohesive soil (dependent on the density index I_D)

Soil type	q^n [kPa] $I_D = 1.00$	q^n [kPa] $I_D = 0.67$	q^n [kPa] $I_D = 0.33$	q^n [kPa] $I_D = 0.20$
Gravel, sandy gravel	7750	5100	3000	1950
Coarse and medium sand	5850	3600	2150	1450
Fine sand	4100	2700	1650	1050
Silty sand	3350	2100	1150	700

The described contemporary basics of dimensioning of piles made by jet-grouting method still leave a lot to be desired. Some doubts should arise about the fact of adapting computational base of load capacity and settlement for piles made of reinforced concrete, included in the standards, textbooks and few papers. Because of its clearly empirical origin it remains on the outskirts of soil mechanics and hence lagging behind present opportunities of numerical modelling and analyzing in this field. Moreover, it is worth noticing that computational values of soil strength below the base and along the shaft, given in the standard PN-83/B-02482 [29] and in the tables 1–4, depend only on the type and state of the soil characterized by parameters resulting from the degree of consolidation or plasticity. The influence of other factors, e.g. geological history expressed by over-consolidation rate OCR, is omitted.

Table 4

Values of unit limiting soil resistance under the pile base q^n [kPa] for cohesive soil (dependent on the liquidity index I_L)

Soil type	q^n [kPa] $I_L < 0$	q^n [kPa] $I_L = 0$	q^n [kPa] $I_L = 0.50$	q^n [kPa] $I_L = 0.75$
Loamy gravel, gravel–sand–clay mix	4150	2750	1650	850
Loamy sand, sandy loam, loam, silty loam	2750	1950	850	450
Firm sandy loam, firm loam, firm silty loam, sandy clay, clay, silty clay	2800	1950	800	400
Sandy silt, silt	1850	1250	500	250

Another objection that can be raised to modern rudiments of jet-grouting piles dimensioning is not-noticing, within the confines of adaptation, their distinction following the technological idea of cement-soil material. This makes impossible to consider in the calculations the dependence of strength and stiffness of pile material on the type, state and history of the surrounding subsoil.

The designer of piles made by jet-grouting method is therefore given the calculation instrument which is not satisfactorily proved and rather conservative in terms of material reserve.

2.2. AIM OF THE COMPUTATIONAL METHODS

The jet-grouting technology brought new sense into classical injection techniques which seemed to be losing their popularity and it became the greatest invention over the last two decades in the field of geoengineering.

It contradicts the computational imperfections of the basics of jet-grouting piles dimensioning presented above.

In the paper, the effort has been made to view the problem in a completely different way – i.e. to approach it within present-day mechanics of continuum, considering co-operation of centers of very different stiffness and their physical non-linearity connected mostly with plastic character of their deformations, especially in the contact zone and in the close soil surrounding of the pile. Such a starting point of the way of thinking brings along a number of consequences in the area of mathematical modelling and computational analysis. They are as follows:

1. Applying heterogeneous model which separates jet-grouting pile, soil massif and contact layer to particular zones.

2. Ascribing adequate constitutive models expressing elastic-plastic behaviour of the materials to particular zones.

3. Using one of the main methods of numerical solving boundary problems of mechanics of continuum – method of finite elements.

The above mentioned postulates provide a quite vast range of opportunities. The simplest idea was chosen for a start. According to it, consideration is aimed at the system of two homogeneous concentric, axially symmetrical bodies, modelling the pile and the soil surroundings and they are divided by contact layer. A simple, elastic-ideally plastic material model is attributed to every of the bodies. The model is described by the flow law associated with boundary condition by Coulomb–Mohr. Such an initial model is then adapted to analysis with the method of finite elements by appropriate digitizing.

Crucial research task is realistic evaluation of the following parameters: modulus of elasticity E_i, Poisson's ratio v_i, internal friction angle ϕ_i and cohesion c_i (i = 1, 2, 3) for every zone. Every one is calibrated separately respecting its specific character.

The experimental base is made up of complex tests of jet-grouting pile, chosen among some made in the test field of Engineering Plant GEOREM in the city of Sosnowiec. The experiments were carried out either in-situ or in laboratories in order to make material and geometrical identification of the jet-grouting pile–soil system, but first of all to provide the basis for verification of the model accuracy and effectiveness.

In case of material of jet-grouting pile, the basis of evaluation is represented by the results of tests of triaxial compression of the samples made of soil-cement material. The tests were conducted in the Laboratory of Rocks Mechanics at the Department of Geomechanics, Underground Engineering and Land Surface Protection at the Faculty of Mining and Geology at the Silesian University of Technology.

In order to estimate soil parameters, some tests in the device of triaxial compression were carried out. They took place in the Laboratory of Soil Mechanics at the Department of Geotechnics at the Silesian University of Technology. On the proving ground of GEOREM penetrating trial loads were performed together with back analysis of the results using finite elements method.

In the light of everything what was considered above, it is possible to make the following proposition: *Numerical model of pile made by jet-grouting method and the proceedings of its calibration are to enable realistic anticipation of the system settlement and of the effort of its material within a wide range of load.*

3. IDEA AND SPECIFICATION OF THE MODEL

3.1. PRELIMINARY REMARKS

According to a general assumption accepted in this paper about evaluation of load capacity and pile made by jet-grouting method, settlement follows the solution of the appropriate originally-boundary problem for the structure–co-operating subsoil system exposed to load from the foundation. The boundary problem is made up of equations of equilibrium (preparatory work), kinematical relations, constituent relations, initial, boundary and contact conditions. Some of the equations have universal character and are common to the deformable body mechanics. The others describe geometry of the system and external field of load (boundary conditions) and mechanical properties of bodies (constituent relations).

Formulating boundary problem requires then defining geometry of the field analyzed and determining its homogeneous zones, specifying static and kinematical influence of the surroundings as well as mathematical model (models) describing behaviour of the pile material and soil and also conditions of zones' interaction.

Specifications listed above form starting point of the analysis of boundary problem. Principles of its realization (numerical one in view of physical-geometric complexity) amount to discontinuing with finite elements and to increment-iterative procedure. They constitute the contents of chapter 3.

3.2. THREE-ZONE STRUCTURE OF THE MODEL

Geometry of the system is determined by jet-grouting pile. Its natural geometrical model is a cylinder, 1 m long and with an average diameter d. In a simple view, force from the foundation has an axial symmetrical effect on the system. In theory, semi-spatial surroundings co-operate with the pile. Classical FEM models of subsoil have limited dimensions. In one's thought, large enough area, symmetrical to the structure, is cut out of the semi-space so that assumed idealization of the boundary conditions on the surface would not disturb significantly stress pattern and displacements in the pile and within its close surroundings.

Standard, in a way, boundary conditions for the cut out solid include:

- conditions for horizontal immobilization and free vertical slip along the side surface,
- complete immobilization of the solid on the bottom surface.

An error of boundary conditions and its impact on the pattern of stress and displacements in the pile and within its close surroundings should be a subject of numerical research. Such a research has shown that a sufficient diameter of the area is $11d$, and the height should be $(l+5d)$, where d – diameter and l – jet-grouting pile length.

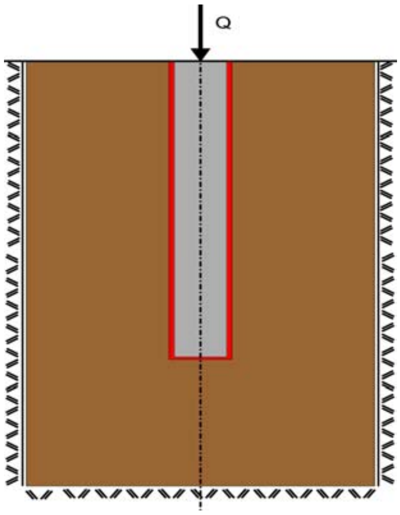

Fig. 2. Geometric model for jet-grouting pile–soil system;
(zone I – jet-grouting pile; zone II – subsoil; zone III – interface layer)

Contemporary research and experiments concerning co-operation of bodies with different stiffness resulted in the idea of thin contact layers. Only an initial geometric model (figure 2) is proposed in this paper.

3.3. MATERIAL MODEL FOR ZONE

3.3.1. GENERAL FRAMEWORK OF MODELLING

Another basic feature of the model proposed for jet-grouting pile is mechanical description of the behaviour of zone materials. It is generally assumed that these are elastic-plastic media described by the law of the same constituent. Only values of parameters are different.

The postulate remains in agreement with the present-day tendency to model soil, rocks and concrete, i.e. the so-called geomaterials, jointly (SANDLER and BARON [31]; CHEN [10]; DESAI and FARUQUE [12]; MRÓZ [25]; ZIENKIEWICZ [34]; BANERJEE et al. [1]; MAJEWSKI [22], [23]). Their distinctive feature is represented by the dependence of shearing resistance on the first stress invariant.

The class of elastic-plastic models of geomaterials is really wide. There are dozens of written studies about the topic available. It is even possible to divide them into three generation groups (GRYCZMAŃSKI [17]):

- elastic-ideally plastic,
- elastic-plastic with isotropic hardening,
- elastic-plastic with isotropic-kinematic hardening.

3.3.2. MODEL SELECTION

At present, selection of the material model for analysis of particular boundary problem of geotechnics seems to be, at least within the country, a compromise between the precision of evaluation of the design quantities (displacements, stresses or section forces) and feasibility. Feasibility means here an access to an appropriate software, technical specifications for material tests, but first of all, difficulty with interpreting results in terms of realistic model calibration. The point is that having to evaluate rationally a set comprising as many as 6–8 material constants which, in addition, do not have physically clear interpretation, we lose control over their properties. Particularly dangerous is lack of information about the sensitivity of parameters of the criteria of adapting results of calculations to changes with applying calibrated model to the real situation. Such a basic criterion can be minimum of the sum of squares of theoretical results deviation from the results of in-situ measurement or measurement in the laboratory. If considerable changes of the parameter correspond with relatively small changes in the sum of squares of deviation that can be easily omitted, it is not possible to evaluate its values with satisfactory precision. This brings into question the adequacy of anticipation with the use of some sophisticated constitutive models.

It seems to be the key argument in favour of a strong tendency which in modern geotechnics is called *experimental soil engineering* (ESE), e.g. DYER et al. [13]. It represents undoubtedly the reaction of those involved in geotechnics to uncontrollable development of constitutive laws. The main idea of the tendency can be summarized in the entry: *simple constitutive models – possibly precise parametric interpretation*.

So, in the light of everything that was considered above, what about a rational ESE model for geomaterials of the jet-grouting pile–soil system?

As an acceptable approximation, elastic-ideally plastic generation, is assumed here. This is justified by the predominant role of shear zone along the shaft of the pile and by possible occurrence of relatively large area of boundary state below its rather small base. Further specification is now trouble-free. Acceptance of the Coulomb–Mohr boundary surface corresponds with the results of the experiments that are far better than Drucker–Prager's approximations, not to mention idealizations by Huber–von Mises–Henky or Tresca. Smooth surface approximations by Coulomb–Mohr described by PODGÓRSKI [30] are even closer to the results of real triaxial tests but they are quite complicated and still not implemented in available FEM packages.

3.3.3. EQUATIONS OF THE COULOMB–MOHR MODEL

In order to present the problem of jet-grouting pile and soil in the area of their cooperation, basic constitutive Coulomb–Mohr model was assumed (COULOMB [11], MOHR [24]). This means that the model response to the load is consistent with Hook's law as long as the state of stress satisfies the inequality:

$$F(\sigma) < 0\,, \tag{7}$$

where stress vector:

$$\sigma = \{\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}\}^T\,. \tag{8}$$

Accomplishing yield criterion $F(\sigma) = 0$ is equivalent to an entry of the material into the boundary state. Exceeding this state is followed by plastic flow and then the inequality $F(\sigma) > 0$ does not describe any real physical state.

Specification of the Coulomb–Mohr model involves:

• Isotropic option of Hook's law

$$\delta\sigma = \mathbf{D} \cdot \delta\varepsilon^e\,, \tag{9}$$

where:

$$\begin{aligned} \delta\sigma &= \{\delta\sigma_x, \delta\sigma_y, \delta\sigma_z, \delta\tau_{xy}, \delta\tau_{yz}, \delta\tau_{zx}\}^T\,, \\ \delta\varepsilon^e &= \{\delta\varepsilon_x^e, \delta\varepsilon_y^e, \delta\varepsilon_z^e, \delta\gamma_{xy}^e, \delta\gamma_{yz}^e, \delta\gamma_{zx}^e\}^T \end{aligned} \tag{10}$$

are the vectors of an increase in effective stress and strain,

$$\mathbf{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2} & 0 & 0 \\ & \text{(sym)} & & & \frac{1-2\nu}{2} & 0 \\ & & & & & \frac{1-2\nu}{2} \end{bmatrix} \tag{11}$$

is the matrix of an isotropic elasticity,

• The Coulomb–Mohr yield criterion; for this model the yield surface, the limiting one and that of boundary state are identical (figure 3) and in the system p, q, Θ they find their expression in the dependence:

$$F(p,q,\Theta) = p\sin\phi - \tfrac{1}{3}q\left(\sqrt{3}\cos\Theta + \sin\Theta\sin\phi\right) + c\cdot\cos\phi = 0\,, \tag{12}$$

but in the isotropic scheme $\left(\Theta = -\dfrac{\pi}{6}\right)$

$$q = \frac{6\sin\phi}{3-\sin\phi}p + \frac{6c\cos\phi}{3-\sin\phi}. \tag{13}$$

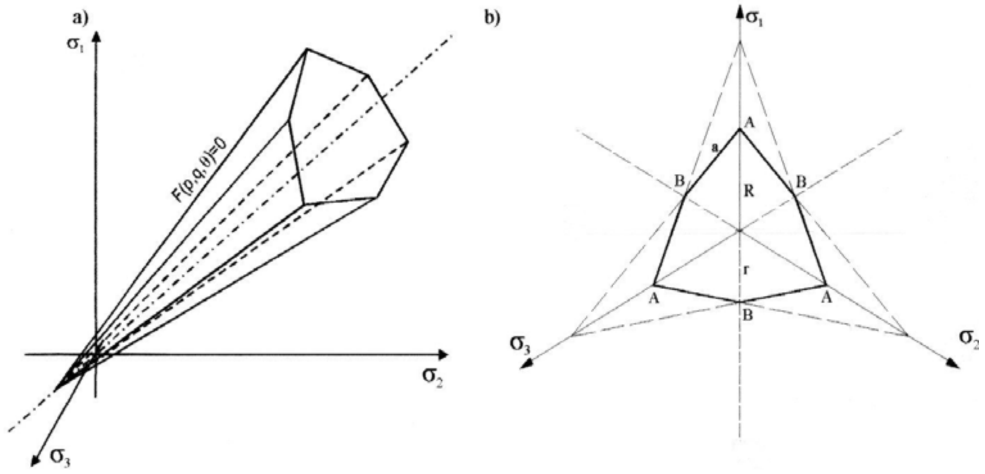

Fig. 3. Yield surface of the Coulomb–Mohr model:
a) acsonometric scheme, b) deviatoric scheme

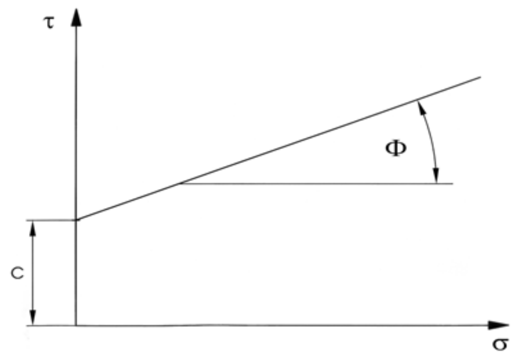

Fig. 4. Graphic interpretation of the Coulomb–Mohr parameters (c and ϕ)

c and ϕ are the parameters of the model and they occur here only, and p, q, Θ are invariants of stress state – average effective stress, stress intensity and Lode's, angle respectively, and they are defined in the following way:

$$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3), \tag{14}$$

$$q = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}, \tag{15}$$

$$\Theta = \arcsin\left\{\frac{1}{\sqrt{2}} \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}}\right\}, \tag{16}$$

and $\sigma_1, \sigma_2, \sigma_3$ are the main stresses.

Parameters c and ϕ have simple physical interpretation which is shown in figure 4.

• Flow law associated with the Coulomb–Mohr yield criterion

$$\delta\varepsilon^p = \lambda \frac{\partial F}{\partial \sigma}, \tag{17}$$

where:

$\delta\varepsilon^p$ – vector of an increase in plastic deformation,

λ – scalar multiplier,

$\partial F / \partial \sigma$ – yield surface gradient (figure 3) determined by the expression:

$$\frac{\partial F}{\partial \sigma} = \frac{1}{3} m \cdot \sin\phi - \frac{1}{2}\frac{s}{q}(\sqrt{3}\cos\Theta + \sin\Theta \sin\phi)$$
$$- \frac{3}{2q^3 \cos 30}\left(q \frac{\partial J_3^D}{\partial \sigma} - \frac{9}{2} J_3^D \cdot s\right)(\sqrt{3}\sin\Theta - \cos\Theta \sin\phi), \tag{18}$$

where:

$$J_3^D = \frac{2}{27}(\sigma_1 + \sigma_2 + \sigma_3)^3 - \frac{1}{3}(\sigma_1^2\sigma_2 + \sigma_1^2\sigma_3 + \sigma_1\sigma_2^2 + \sigma_2^2\sigma_3 + \sigma_1\sigma_3^2 + \sigma_2\sigma_3^2) \tag{19}$$

is the third invariant of deviator of stress tensor,

$$m = \{1, 1, 1, 0, 0, 0\}^T \tag{20}$$

is a unitary vector, and,

$$s = \sigma - \frac{1}{3} m p \tag{21}$$

is a deviator of stress vector.

After having taken into account the constitutive laws and additivity principle $\delta\varepsilon = \delta\varepsilon^e + \delta\varepsilon^p$ and the condition of impassability of yield surface $(\partial F / \partial \sigma)^T \cdot \delta\sigma = 0$, and after some transformations, there is obtained a general constitutive relation for elastic-ideally plastic models, which is a special case of the equation of elastic-plastic matrix:

$$\delta\sigma = \left[D - \beta \frac{D \cdot \left\{\frac{\partial F}{\partial \sigma}\right\} \cdot \left\{\frac{\partial F}{\partial \sigma}\right\}^T \cdot D}{\left\{\frac{\partial F}{\partial \sigma}\right\}^T \cdot D \cdot \left\{\frac{\partial F}{\partial \sigma}\right\}} \right] \cdot \delta\varepsilon \tag{22}$$

when $\beta = 0$ for $F(\sigma) < 0$ and $\beta = 1$ for $F(\sigma) = 0$.

The above-mentioned expressions are followed by the conclusion that the accepted Coulomb–Mohr model includes four parameters: internal friction angle ϕ, cohesion c, modulus of elasticity E and Poisson's ratio ν.

3.4. NUMERICAL REALIZATION (DISCONTINUING FEM, COMPUTER PROGRAMS)

For the described above three-zone model co-operating with the subsoil of the jet-grouting pile, the method of finite elements was applied.

The co-operating system represents three-dimensional, axially symmetrical problem, i.e. arithmetically flat problem. The material zones analyzed were made discrete, i.e. they were parted and the boundaries of the material zones were determined as boundaries of the elements. Each of the material zones of the jet-grouting pile and soil was additionally divided, while a contact zone is regarded as a layer of one-element thickness. Block of the model was considered (see table 5) with the diameter of 3.30 m {i.e. 11*d*} and the height of 3.80 m {(*l* + 5*d*)}, in which three material zones were dealt with. The area was divided into eight-noded, quadrilateral, isoparametric elements.

Shape of the system of finite elements is adjusted to the geometry of the pile (see figure 5 & photo 1) and soil. The further from the point of contact of the pile and the subsoil, the bigger the dimensions for the elements used.

Table 5

Profile of the mesh modelled for jet-grouting pile–soil system

Number of super-elements	Number of super-nodes	Number of elements in the mesh	Number of main nodes in the mesh	Number of elements for removing for preparing original mesh	Mesh dimensions [m] × [m]
10	18	361	346	75	3.80 x 1.65

Remark: Super-nodes and super-elements are the conveniences of CRISP'93 program in the range of mesh generation.

The boundary conditions were assumed as typical of the problems of soil mechanics. They are node pivot-sliding bearings along the vertical axis of symmetry and vertical side-edge, as well as node pivot-nonsliding bearings for the bottom edge of the model.

Analysis by the method of finite elements can be carried out with the aid of one of the available program packages which include the Coulomb–Mohr model, i.e. Z_SOIL, PLAXIS, CRISP, HYDROGEO can be used as well as some others. For the numerical analysis CRISP'93 was used (BRITTO and GUNN [3], [4], [5]). It was developed as a program introducing critical state models (that's why it is called **Cri**tical **S**tate **P**rogram).

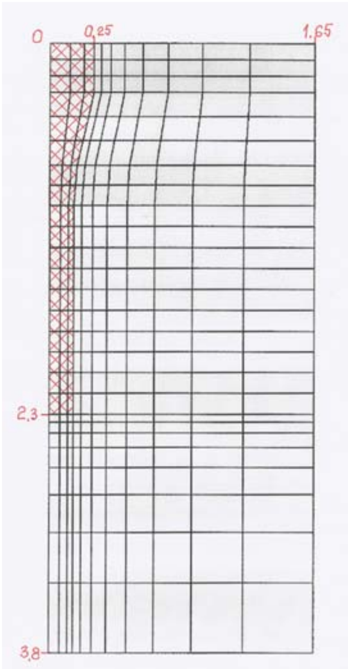

Fig. 5. The finite-elements mesh for numerical analyses

Photo 1. Jet-grouting pile after excavation and cleaning at the laboratory

CRISP'93 program uses the increment-iterative method which enables us to update the results after every increment. So the process of loading the pile proceeded by increments.

4. MODEL CALIBRATION

This chapter deals with the calibration of three-zone model, i.e. with evaluation of the parameters of the Coulomb–Mohr model for the material of jet-grouting pile, soil co-operating with the pile – medium sand and silty clay as well as contact zone.

The research proceedings, instruments and criteria of calibration were adapted to the specificity of the materials. Cement-soils, which form the material of the pile, were tested at the Laboratory of Rocks Mechanics in the high-pressure apparatus for triaxial compression, according to standard loading programs (at a constant chamber pressure). Modulus of elasticity and Poisson's ratio were estimated on the grounds of adjustment of linear characteristics $q - \varepsilon_1$ and $q - \varepsilon_3$, and internal friction angles and cohesion – on the grounds of adjustment of the Coulomb–Mohr straight lines to the results of strength tests.

In order to evaluate the modulus of elasticity, internal friction angle and cohesion of the model for the material of the pile, a series of standard tests was carried out in the high-pressure apparatus of triaxial compression, on the samples cut out of the stem of the jet-grouting pile (BZÓWKA, [8], [9]). Such tests of material of jet-grouting piles have not been carried out so far. Therefore, apart from collecting database, the results obtained have also cognitive value.

They justify the following conclusions:

• modulus of elasticity and cohesion of soil-cements are, respectively, 5–8 times and twice as small as average values of those parameters for concrete,

• modulus of elasticity of the jet-grouting pile material is insensitive to changes of side stress,

• the influence of the soil being injected on the parameter values is not big although clearly noticeable.

It has to be made it clear that the above-mentioned quantity relations correspond with pressure of injection that was used to make jet-grouting pile.

Much more serious problem was connected with realistic evaluation of the model parameters such as material constants of soil surroundings layers, since the dependence of the strength parameters on the conditions of drainage and particular sensitivity of modulus of elasticity to stress path were known.

So the experiments were conducted according to two fundamentally different programs. Within the first one, triaxial test was carried out, similar to that in the case of pile material, but together with reconstruction of the history in the form of anisotropic reconsolidation of K_0 type and with water pressure measurement in the shear phase "without drainage". Undisturbed samples of silty clay out of the depth of 4.5 m were tested. Effective values of the parameters were evaluated. In the case of estimation of modulus of elasticity, the well-known interpretation difficulties occurred. They resulted from strong nonlinearity and sensitivity to side stress of the characteristics $q - \varepsilon_1$. An effort was made to overcome them by: samples' reconsolidation up to the state of initial stresses, keeping the achieved side stress in the shear phase and recommended in writings averaging of $E_{25\%}$. The value obtained bears some uncertainty which is due to the fact that during the pile loading, change of stress path was not taken into account and that the definition of averaging was arbitrary and because of dissipation of the results with a too small sample.

Therefore, another, additional realization of the second program was justified. The program is called *penetrating trial load* and is numbered among the procedures of the so-called global calibration (GRYCZMAŃSKI [16]). The criterion there was such a selection of the model parameters that theoretical load–settlement characteristic of the shore acting in cylindrical hole inside the soil massif was optimally adjusted to the results of penetrating trial load at a given depth.

The idea of penetrating trial load has been well-known, but the description and results of its specific applications have only been met by the author in the papers from her own department (cf., e.g., PIECZYRAK [27]).

The disadvantage of conducting the penetrating trial loads by the method of constant steps of loading seems to be the fact that at every step a consolidation process is initiated but generally not completed. So, in order to evaluate effective values of parameters, it would be necessary to reverse the problem of consolidation (cf. e.g. GASZYŃSKI [15]). On the other hand, this approach has a big advantage in the context of model verification. There is space-time similarity of test conditions with verifying trial load of jet-grouting pile. Keeping this in mind, estimations obtained in this way were assumed as authoritative.

Another problem is evaluation of contact zone parameters. In the paper, it is not thoroughly considered. The simplest solution was accepted – i.e. proportional reduction of values evaluated for soil massif. Estimation of the contact zone parameters is based on the assumption that they are reduced (by 33%) parameters for subsoil.

5. MODEL VERIFICATION AND ANALYSIS

5.1. PRELIMINARY REMARKS

The idea and mathematical description of the theoretical model of the jet-grouting pile were presented in chapter 3, and its parametric identification in chapter 4. Now it is time to check justification of either the choice or model effectiveness and thus to prove legitimacy of the proposition presented in chapter 2.

Comparison of theoretical and experimental characteristics of axial force–settlement of jet-grouting pile is to serve as a tool to achieve this aim. The experimental database for the sake of comparison is made up by the results of trial load of a selected jet-grouting pile. Theoretical characteristic is determined by the results of FEM analysis of three-zone model of the pile, with parameters evaluated separately for each zone.

Experimental verification is to be presented from the description of the course and results of trial load, through numerical simulation, as to specification and assessment of the results of confrontation.

In chapter 5.3, there will be presented a comparison of the results of the analyses conducted according to existing propositions and according to the results of FEM analysis with application of the three-zone model.

The last stage of proving model effectiveness is the analysis of its sensitivity (see chapter 5.4).

5.2. VERIFICATION OF EXPERIMENTAL MODEL

Experiments to test trial load of jet-grouting pile were carried out on the primary load, and then, after unloading, on the secondary load. Primary load was done in twelve steps of load. Value of the following load was connected with the possibility of precise reading of load value showed by manometer. It was 0.2 MPa or 0.3 MPa alternately. So the following steps were: 0.5; 0.7; 1.0; 1.2; 1.5; 1.7; 2.0; 2.3; 2.6; 3.0; 3.6; 4.0 MPa. Using the characteristics of manometer calibration, the value of vertical force applied was achieved for each of the steps and its maximum was 215 kN. Measurement of settlement was recorded for each step of loading every 2 minutes, until the moment of readings stabilization, i.e. when the difference between the following readings was smaller than 0.05 mm per 10 minutes. Graphic interpretation of the test is shown in figure 6.

5.2.1. COMPARISON OF THE RESULTS OF MEASUREMENTS AND CALCULATIONS

The results of trial load of the experimental jet-grouting pile are to be compared with theoretical characteristics of loading–settlement of its three-zone model, corresponding with tests conditions.

Fixing this characteristics requires, generally speaking, solving axially-symmetrical boundary problem for stepwise heterogeneous, elastic-ideally plastic body being comprised of jet-grouting pile, contact layer, cylindrical subsoil solid and soil surroundings. In the light of chapter 3.4, solution to such a problem cannot have another form but increment-iterative numerical analysis; moreover, the method of finite elements is unbeatable in this case. In chapter 3.4, general principles of creating FEM model for the jet-grouting pile–soil system with contact layer as well as recommendations concerning model dimensions, type of elements and geometry of mesh were presented.

Ultimately, what is used for the present analysis is the discrete scheme (figure 5) and the parameters of following zones presented in tables 6a–6b.

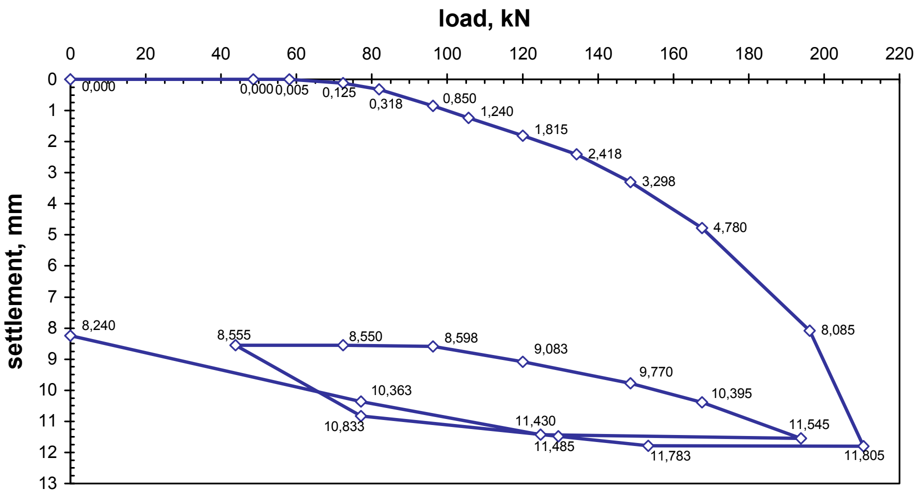

Fig. 6. Relation of load–settlement – experimental curve of jet-grouting pile

Table 6a

The Coulomb–Mohr parameter for jet-grouting pile

Zone	Parameter quantity	Material parameter	
		Cement-clay mix	Cement-sand mix
Jet-grouting pile	Modulus of elasticity E_1 [MPa]	4720	5630
	Poisson's ratio ν_1 [1]	0.20	0.17
	Friction angle ϕ_1 [°]	51.8	55.8
	Cohesion c_1 [kPa]	3530	3640

Table 6b

The Coulomb–Mohr parameter for subsoil

Zone	Parameter quantity	Parameters	
		Silty clay, $I_L = 0.10$	Medium sand, $I_D = 0.50$
Subsoil	Modulus of elasticity E_2 [MPa]	$64.0 + 11.84 \cdot (z - 4.75)$	108.9
	Poisson's ratio ν_2 [1]	0.20	0.25
	Friction angle ϕ_2 [°]	9.8	28.5
	Cohesion c_2 [kPa]	$30.0 + 4.0 \cdot (z - 4.75)$	2.5

Table 6c

The Coulomb–Mohr parameter for interface zone

Zone	Parameter quantity	Parameters of interface zone	
		In silty clay layer	In medium sand layer
Interface layer	Modulus of elasticity $E_3 = 0.67E_2$ [MPa]	42.7	72.6
	Poisson's ratio $\nu_3 = \nu_2$ [1]	0.20	0.25
	Friction angle $\phi_3 = 0.67\phi_2$ [°]	6.5	19
	Cohesion $c_3 = 0.67c_2$ [kPa]	20	1.7
	Thickness of interface zone t [m]	0.0005	0.0005

Starting point for the numerical analysis was the state of in-situ stress. Simulation of trial load of jet-grouting pile consisted in increasing load successively (1000 steps of increase) until the maximum value was reached.

FEM analysis is focused on specifying the dependence between average unitary load acting on the pile head and vertical displacement of the head midpoint which is tantamount to pile settlement. An adequate pressure–settlement characteristics within the range of vertical loads is presented in figure 7a and compared with experimental curve.

The best adjustment of theoretical curve to the experimental one is shown in figure 7b.

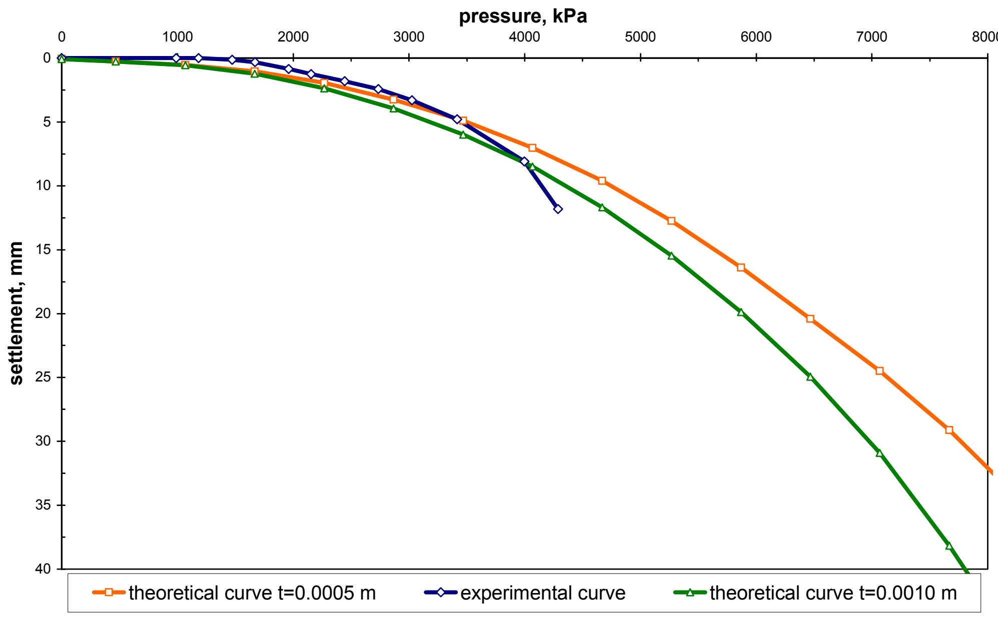

Fig. 7a. The best adjustments of the theoretical curve to the experimental curve

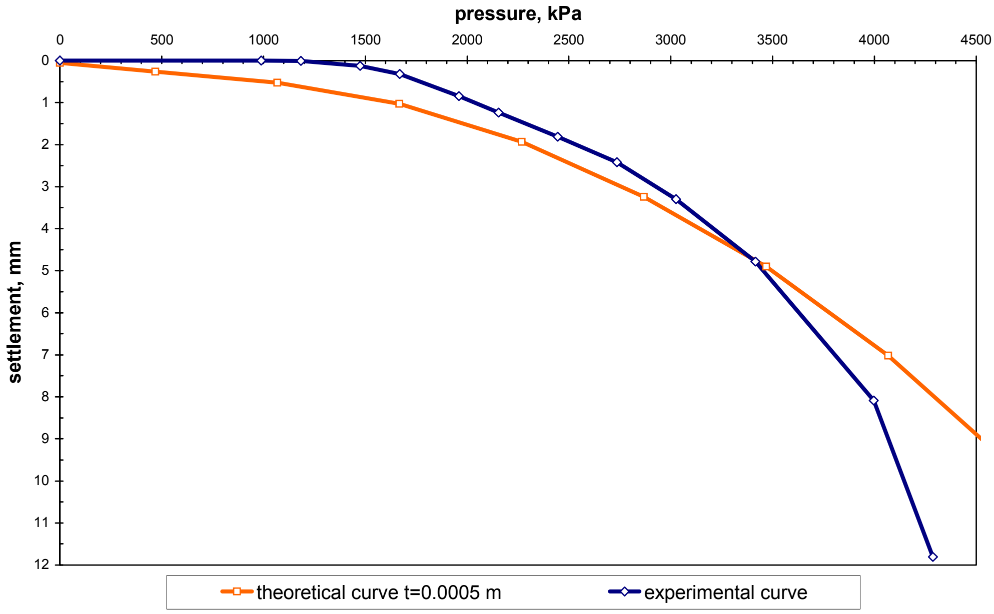

Fig. 7b. The best adjustment of the theoretical curve to the experimental curve
(cf. figure 7a in the changed scale)

Real response of soil massif to the loads, reconstructed within the confines of different kinds of measurements, are characterized by smaller or bigger irregularities and anomalies. A heterogeneous character of soil structures and their changes in the process of loading seem to be their main sources.

For that reason, tolerance for discrepancy between measurement results and calculations with application of particular computational models is definitely greater in geotechnics than in reinforced concrete or metal engineering.

Keeping this in mind, it should be stated that the consistency of theoretical and experimental characteristics in figure 7a is very good within almost all the range of trial loads. Only within short, final section of the characteristics a discrepancy appears – the settlements measured grow with the load quicker than those calculated. Therefore, considerable caution must be recommended while estimating model quality beyond the trial load range.

To be able to say something more about the topic it will be necessary to have larger database of the results of trial loading conducted until limiting loading capacity state of the system and all the supporting experiments, essential for model calibration.

Irrespective of such a protection, with a view of real settlements progression, a possibility of applying more sophisticated models, at least for subsoil, has to be considered.

The existing numerical studies (ZIENKIEWICZ and HUMPHESON [35]; ZIENKIEWICZ et al. [36]) show that the simplest move will be introducing, in place of being used now, associated flow law, non-associated flow law based on application of dilatation angle $\psi = 0$ (cf. figure 8).

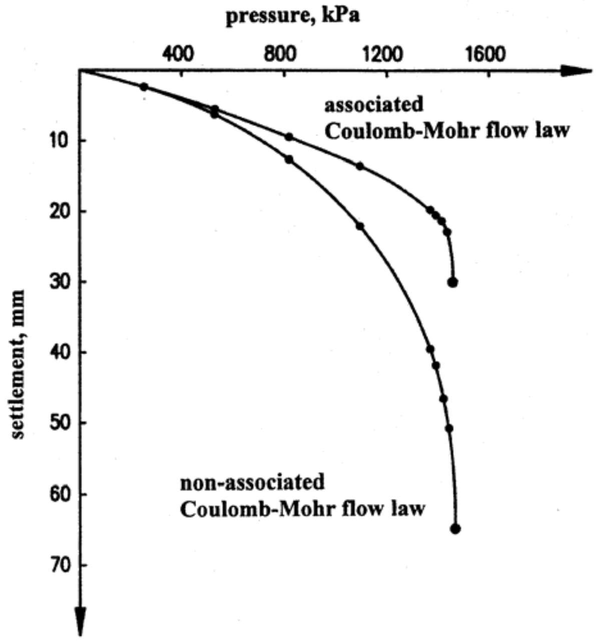

Fig. 8. Results of the bearing capacity analyses with using associated and non-associated flow laws (ZIENKIEWICZ and HUMPHESON [35])

Further search should be directed towards subsoil description (and contact layer) by the elastic-plastic law with isotropic reinforcement, which belongs to the family of models of critical state, e.g. Modified Cam-clay (BURLAND [6]), or Roscoe–Hvorslev (HOULSBY et al. [19]), or to the models family "cap", e.g. "double cap" (MAJEWSKI [22], [23]).

5.3. COMPARING THE MODEL WITH THE EXISTING PROPOSITIONS

Quite obvious and important element of verification of every new computational model is its confrontation with the existing ones. Within the scope of such confrontations efforts are made to answer the question whether the results of calculations obtained with the use of comparable models are convergent or significantly different. In the former case confirmation of validity of the existing computational base is achieved, the latter points out that it is reasonable to search for new solutions.

Unfortunately, the possibilities of comparing three-zone model of the jet-grouting pile–soil system, developed in the paper, with schemes presented in chapter 2.1 are relatively small. On the one hand there is a theoretical model created within the theory of plasticity which proved useful in a wide range of load, but is was not verified in the aspect of limiting bearing capacity. Extrapolation of the results of trial load seems to be risky, also anticipating boundary bearing capacity with the use of elastic-ideally plastic model together with associated flow law. On the other hand there are semi-empirical models of limiting state which describe exclusively limiting bearing capacity. Theory is represented there by the condition stating that vertical force applied axially to the top surface of the pile is balanced by forces of soil massif response – component tangent of stress on the surface of base. Average values of the components are determined empirically.

Comparing the model developed in the paper with the semi-empirical models of limiting state mentioned above, it is only possible to compare limiting bearing capacity, anticipation of which by the former one is encumbered with considerable uncertainty.

Much more reasonable would be comparing a new model with the proposal by GWIZDAŁA and MOTAK [18] in the aspect of predicting pile settlement as a function of load; but there is lack of data concerning parameters of functions of transformation for the pile tested.

Hence, there is nothing to do but stay satisfied with comparison of estimations of limiting load obtained with the use of different models (table 7). They give only information about general trend.

Table 7

Comparison of the value of limit load for three-zone model with the existing propositions

Value of limit load [kN]	According to Polish Pile Code PN-83/B-02482 [29]	According to ŻMUDZIŃSKI and MOTAK [38]	According to GWIZDAŁA and MOTAK [18]	The model proposed (for t = 0.0010 m)
	249	264	362	565

First three values were calculated on the grounds of formulae (2), (4), (6) and tables 1–4 for the layers of:

- medium sand I_D = 0.50, for the surface of contact with the pile A_{s1} = 0.518 m^2,
- silty clay I_L = 0.10, for the surface of contact with the pile A_{s2} = 2.123 m^2.

Computational base surface was determined as A_p = 0.071 m^2.

Having examined table 6 the following conclusions can be formulated:

1. Every next proposal is characterized by a greater bearing capacity.
2. From figure 7 it follows that real limiting bearing capacity of jet-grouting pile is placed between GWIZDAŁA and MOTAK's prediction [18] and the value anticipated on the basis of three-zone model.

3. The latter value is, in fact, estimated with excess and it has been already mentioned; therefore the need to improve the model of soil massif, e.g. towards non-associated flow law, has been confirmed.

4. Estimation by GWIZDAŁA and MOTAK [18] can be regarded as satisfactory approximation but with some insufficiency.

5.4. ANALYSIS OF MODEL SENSITIVITY

5.4.1. PROGRAM OF THE ANALYSIS

Results of the analysis of mathematical model sensitivity to changes of parameters are to a large extent determined by its effectiveness and utility value. Parameters estimated on the basis of experimental tests are often afflicted with significant measurement uncertainties. In case of sensitive models, they generate even greater uncertainties of predictions about their response to the external influence.

The dependences of global mechanical areas (displacement, contact stresses) within the three-zone, elastic-plastic model of jet-grouting pile on parameters are nonlinear and complex. As it was stated in chapter 5.1, only numerical analysis of sensitivity based on sets of parameters values is possible. Complex test of the relation between independent vector field of parameters and dependent vector field of all state variables or state characteristics would not be practically feasible.

The program assumed in this paper is by far more humble. The object of numerical experiments is sensitivity of the pressure–settlement characteristic of the pile to changes of modulus of elasticity, internal friction angles and cohesion of pile's cement-soil, the surroundings, subsoil and contact zone. The influence of contact zone thickness is tested independently.

In a series *j* of tests, only one parameter changes. It is generally marked as x_j and is a component of vector

$$\{x_j\}_{j=1,\ldots,9} \stackrel{\mathrm{df}}{=} \{E_1, c_1, \phi_1; E_2, c_2, \phi_2; E_3, c_3, \phi_3\}\,.$$

It takes the following values: $0.25\hat{x}_j$, $0.5\hat{x}_j$, $1.5\hat{x}_j$, $2\hat{x}_j$, where $\hat{x}_j$ is an estimation x_j for the jet-grouting pile–soil system. So the maximum is always eight times bigger than the minimum. The other parameters x_k ($k \neq j$) do not undergo any changes in the *j* series and their values are equal to estimations $\hat{x}_k$ for the jet-grouting pile–soil system.

For every combination of this type, simulation FEM of trial load is carried out and theoretical characteristic of load–settlement appointed.

The model sensitivity to changes of contact layer material parameters is examined for two of its thickness.

A schedule of changes of model material parameters in the following series of numerical tests comprising the analysis of sensitivity is shown in table 8.

The study is complemented by the research of contact layer influence on the course of characteristics $\overline{q} - s$. The FEM analysis was carried out at eleven thicknesses ranging from 0.1 mm to 5 mm (table 9), assuming that the values of material parameters are estimations for the jet-grouting pile–soil system.

Table 8

Program of the changes of consecutive material parameters in the analysis of sensitivity of the pressure – settlement characteristic for jet-grouting pile–soil system

Number of numerical analyses	Changeable model's parameter	Values				Thickness of the interface layer
		$0.25\hat{x}_j$	$0.5\hat{x}_j$	$1.5\hat{x}_j$	$2\hat{x}_j$	
1	E_1 [MPa]	1180	2360	7080	9440	t = 0.0005 m
2	c_1 [kPa]	883	1765	5295	7060	
3	ϕ_1 [°]	13	25.9	77.7	–	
4	E_2 [MPa]	16	32	96	128	
5	c_2 [kPa]	–	15	45	60	
6	ϕ_2 [°]	2.5	4.9	14.7	19.6	
7	E_3 [MPa]	10.7	21.3	64	85.3	t = 0.0005 m
8	E_3 [MPa]	10.7	21.3	64	85.3	t = 0.0010 m
9	c_3 [kPa]	5	10	30	40	t = 0.0005 m
10	c_3 [kPa]	5	10	30	40	t = 0.0010 m
11	ϕ_3 [°]	1.6	3.3	9.8	13	t = 0.0005 m
12	ϕ_3 [°]	1.6	3.3	9.8	13	t = 0.0010 m

Table 9

Program of the changes of the thickness of contact layer in the analysis of sensitivity of the pressure–settlement characteristic for jet-grouting pile–soil system

Coulomb–Mohr parameters	Values of the parameters			
	For jet-grouting pile	For subsoil	For interface zone	Changeable parameter – thickness of the interface layer t [m]
E_i [MPa]	4720	64	42.7	t = 0.0001 t = 0.0002 t = 0.0003 t = 0.0004 t = 0.0005 t = 0.0006 t = 0.0007 t = 0.0008 t = 0.0009 t = 0.0010
c_i [kPa]	3530	30	20	
ϕ_i [°]	51.8	9.8	6.5	

				$t = 0.0050$

Diverging from the rule of changing only one parameter in one series, there is additionally analyzed the case of proportional changes $\{x_m\}=\{E_3, c_3, \phi_3\}$ creating the set: $0.25\,\{\hat{x}_m\}$, $0.50\,\{\hat{x}_m\}$, $0.67\,\{\hat{x}_m\}$, $0.75\,\{\hat{x}_m\}$, $\{\hat{x}_m\}$ (table 10).

Table 10

Program of simultaneous material parameters of contact layer in the analysis of sensitivity of the pressure–settlement characteristic for jet-grouting pile–soil system

Changeable model's parameter	Values of the parameters					Thickness of the interface layer
	$0.25\{\hat{x}_m\}$	$0.50\{\hat{x}_m\}$	$0.67\{\hat{x}_m\}$	$0.75\{\hat{x}_m\}$	$\{\hat{x}_m\}$	t [m]
E_1 [MPa]	4720	4720	4720	4720	4720	$t = 0.0005$ $t = 0.0010$
c_1 [kPa]	3530	3530	3530	3530	3530	
ϕ_1 [°]	51.8	51.8	51.8	51.8	51.8	
E_2 [MPa]	64	64	64	64	64	
c_2 [kPa]	30	30	30	30	30	
ϕ_2 [°]	9.8	9.8	9.8	9.8	9.8	
E_3 [MPa]	16	32	42.7	48	64	
c_3 [kPa]	7.5	15	20	22.5	30	
ϕ_3 [°]	2.5	4.9	6.5	7.4	9.8	

5.4.2. RESULTS AND CONCLUSIONS

Results of the analysis of model sensitivity are presented in figures 9–24. Each figure includes a pressure–settlement curves corresponding with values of another parameter. As is known, the values remain in particular ratio (bigger or smaller than one) to the estimation of given parameter of the jet-grouting pile model.

Corresponding curves $\overline{q}-s$ are placed on both sides of the jet-grouting pile theoretical characteristics.

Measures of model sensitivity to the parameter can be various. What can be a convenient measure is the change of the value of pile settlement with increasing or decreasing the value of parameter in relation to optimum estimation by fixed proportional value, e.g. by 50%. The change is obviously an increasing function of load which can be presented with the aid of analytical formula or scheme.

Sensitivity of the pressure–settlement characteristic of the pile model to an increase in the value of parameters E_j, c_j, ϕ_j (j = 1, 2, 3) by 50% is shown in figure 25, and to a decrease by the same proportion – in figure 26.

It follows from figures 9, 10 and 11 that in terms of settlement the model is completely insensitive to the changes of pile parameters. It is confirmed by the curves of sensitivity to the parameters E_1, c_1, ϕ_1 in figures 25 and 26, which practically agree with the load axis.

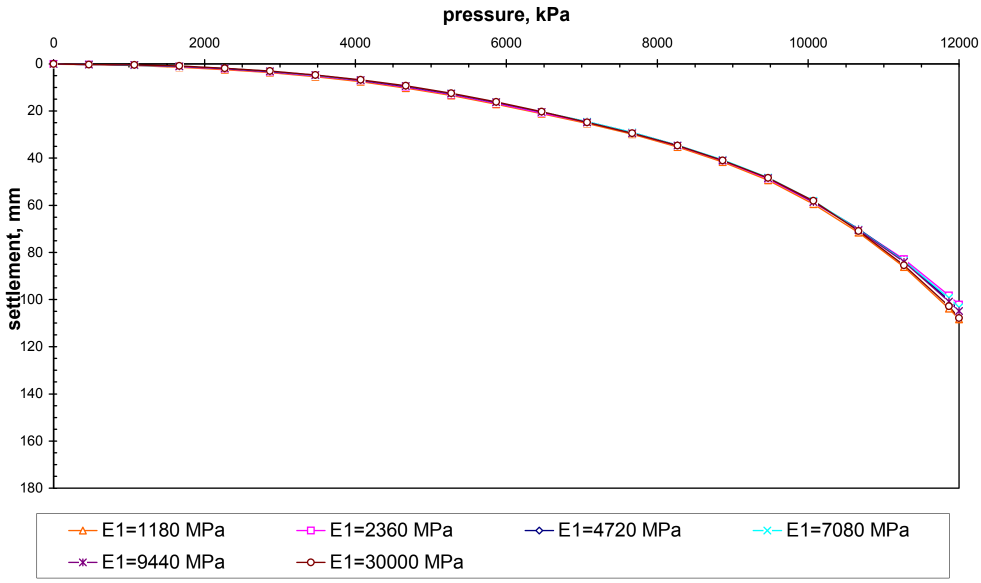

Fig. 9. Relation of a pressure–settlement as a function of modulus of elasticity E_1 of jet-grouting pile

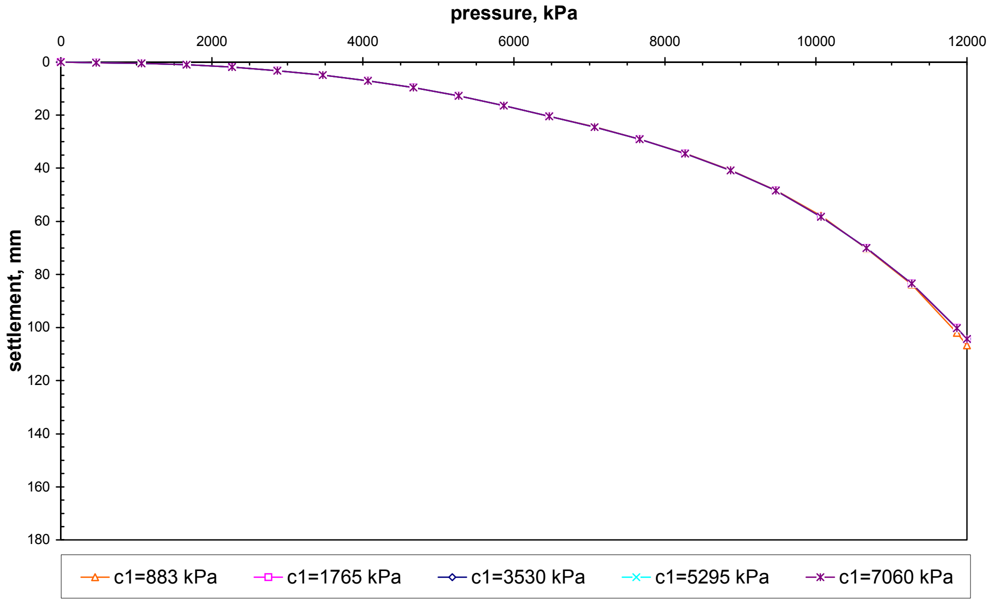

Fig. 10. Relation of pressure–settlement as a function of cohesion c_1 of jet-grouting pile

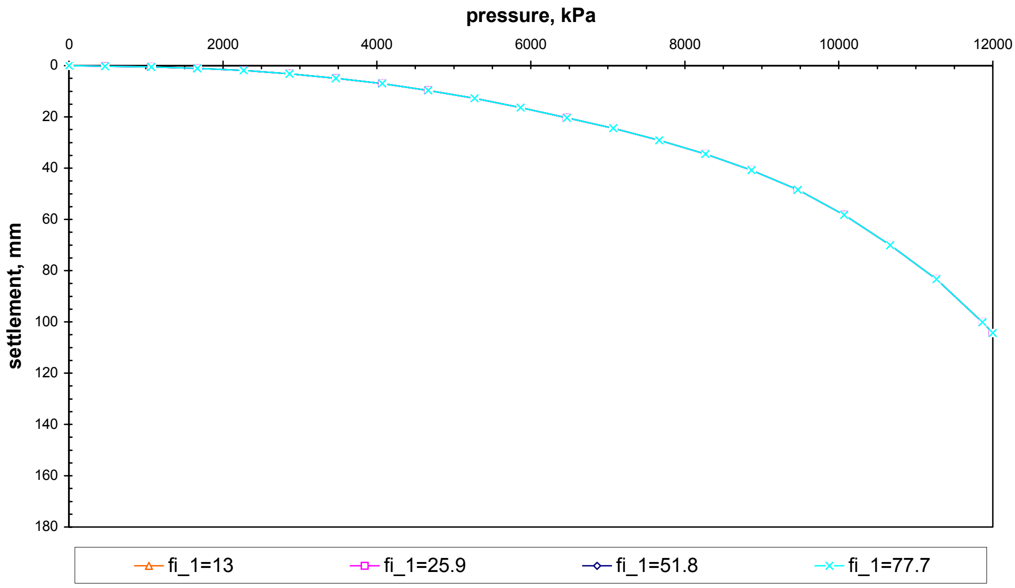

Fig. 11. Relation of pressure–settlement as a function of internal friction angle ϕ_1 of jet-grouting pile

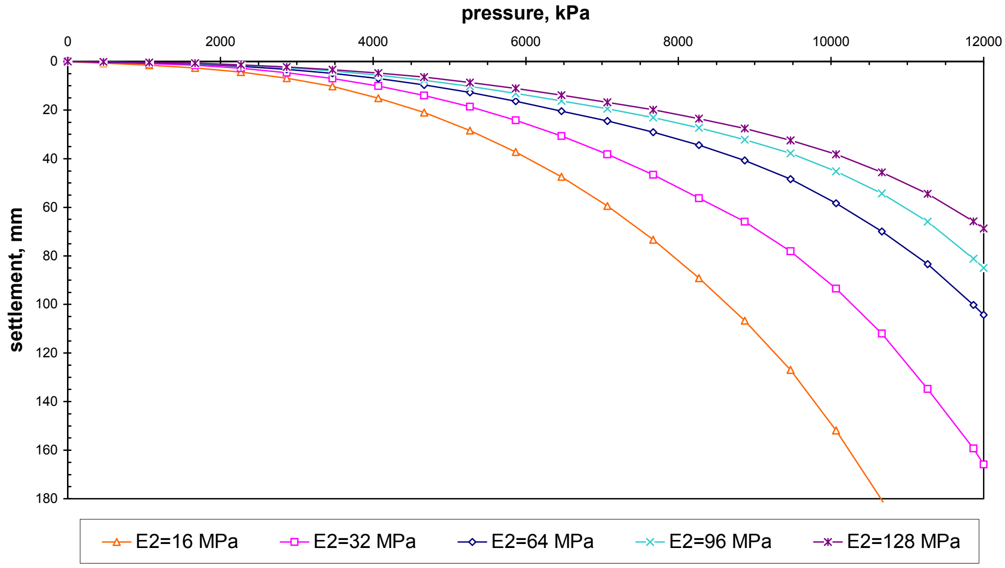

Fig. 12. Relation of pressure–settlement as a function of modulus of elasticity E_2 of silty clay

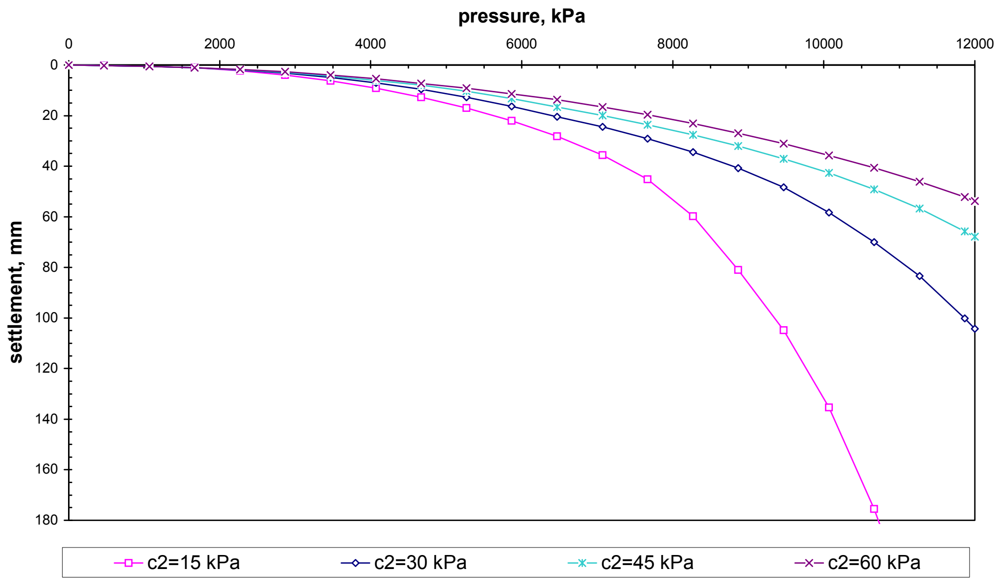

Fig. 13. Relation of pressure–settlement as a function of cohesion c_2 of silty clay

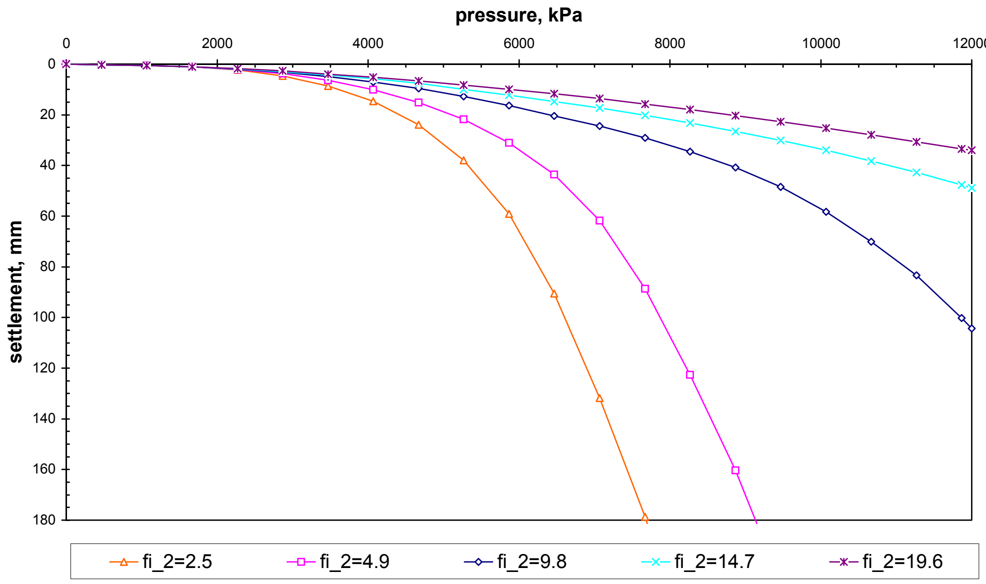

Fig. 14. Relation of pressure–settlement as a function of internal friction angle ϕ_2 of silty clay

The analysis of figures 12, 13 and 14 as well as figures 25 and 26 leads up to some essential comments:

a) *q–s* characteristics of the model, more weakly but in a visible way, responds to an increase and quite strongly to a decrease in subsoil parameters,

b) model is the least sensitive to the changes of modulus of elasticity and the most to the changes of internal friction angle,

c) similar situation is with the tendency of sensitivity to increase at an increase in load, particularly strong is progression of settlements with load in case of a decrease in internal friction angle, which is slightly noticeable at an increase in modulus of elasticity.

Figures 15–20 and 25–26 show sensitivity of the characteristics *q–s* of the pile to the change of material parameters of contact layer. Numerical experiments were conducted simultaneously at two zone thicknesses, i.e. 0.5 and 1.0 mm. Considering the fact that the thickness is quite small, its influence on the pile settlement should be regarded as significant.

It follows from the experiments that:

a) model sensitivity to the changes of material parameters of contact zone is bigger when the layer is 1.0 mm thick than when it is 0.5 mm thick,

b) model characteristics *q–s* is the least sensitive to the changes of modulus of elasticity and the most – to the changes of internal friction angle,

c) unlike the case of subsoil the influence of a given parameter increase is more significant than that of a decrease.

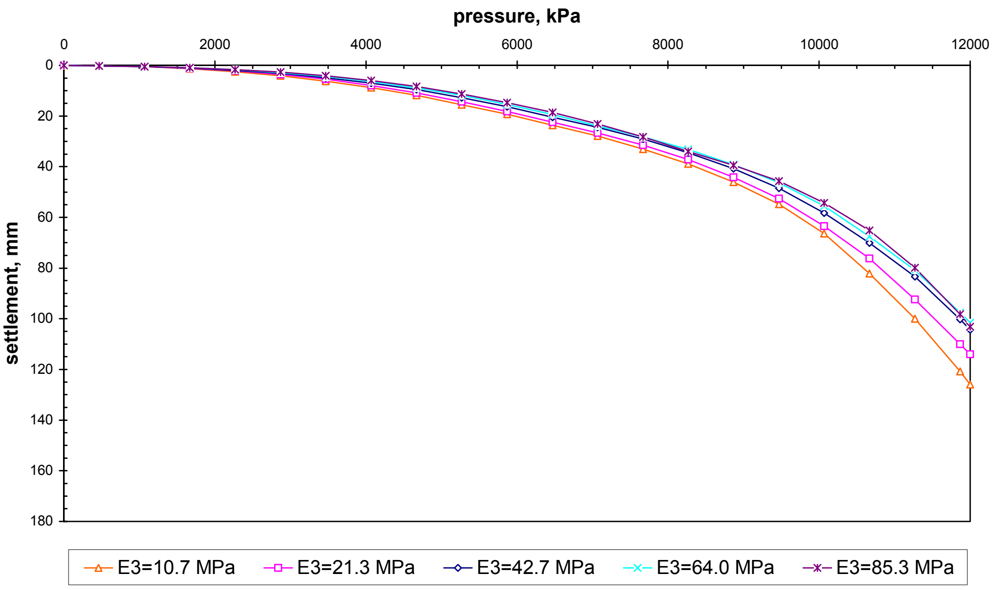

Fig. 15. Relation of pressure–settlement as a function of modulus of elasticity E_3 of interface layer (for $t = 0.0005$ m)

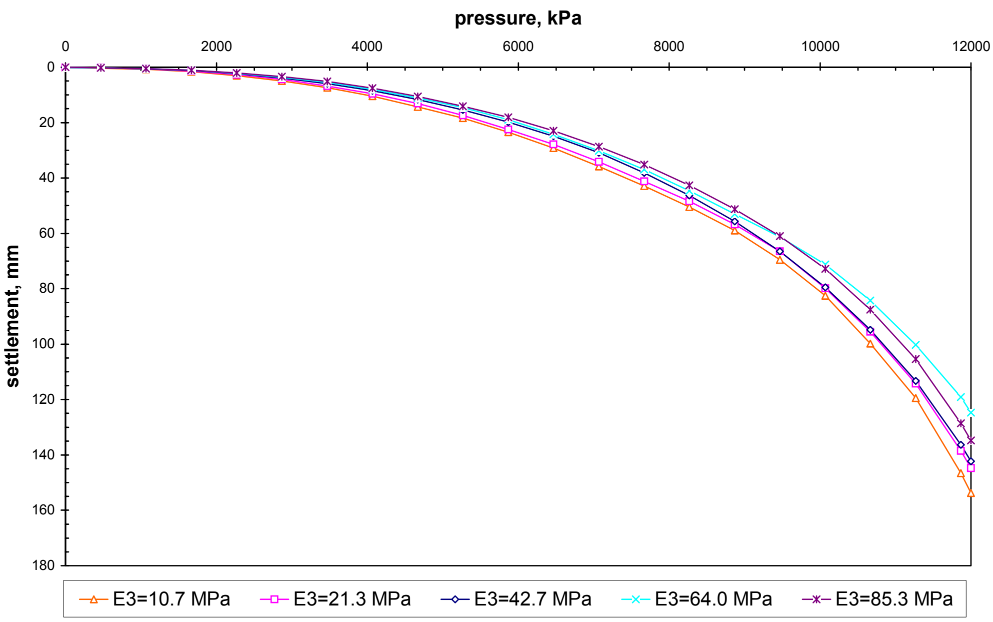

Fig. 16. Relation of pressure–settlement as a function of modulus of elasticity E_3 of interface layer (for t = 0.0010 m)

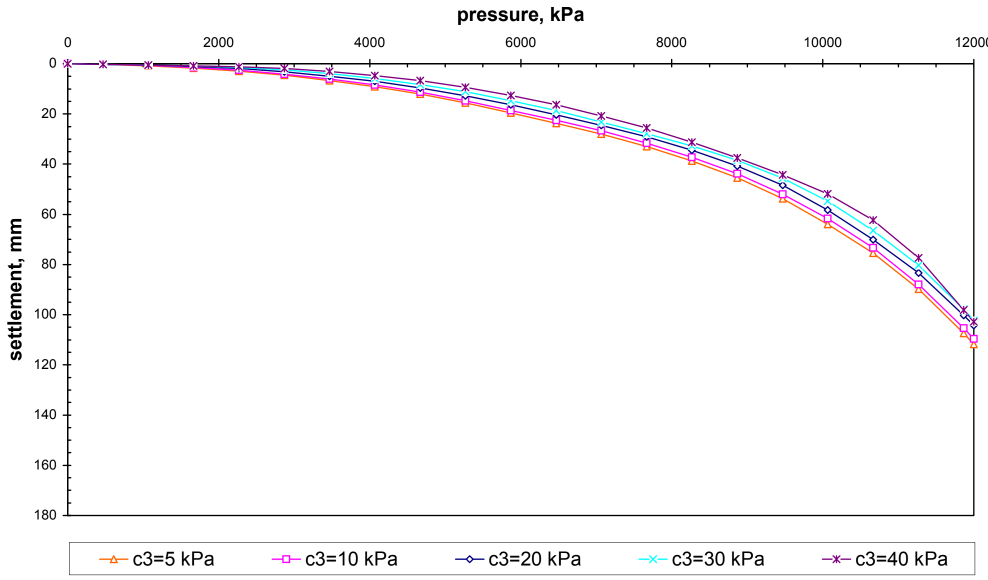

Fig. 17. Relation of pressure–settlement as a function of cohesion c_3 of interface layer (for t = 0.0005 m)

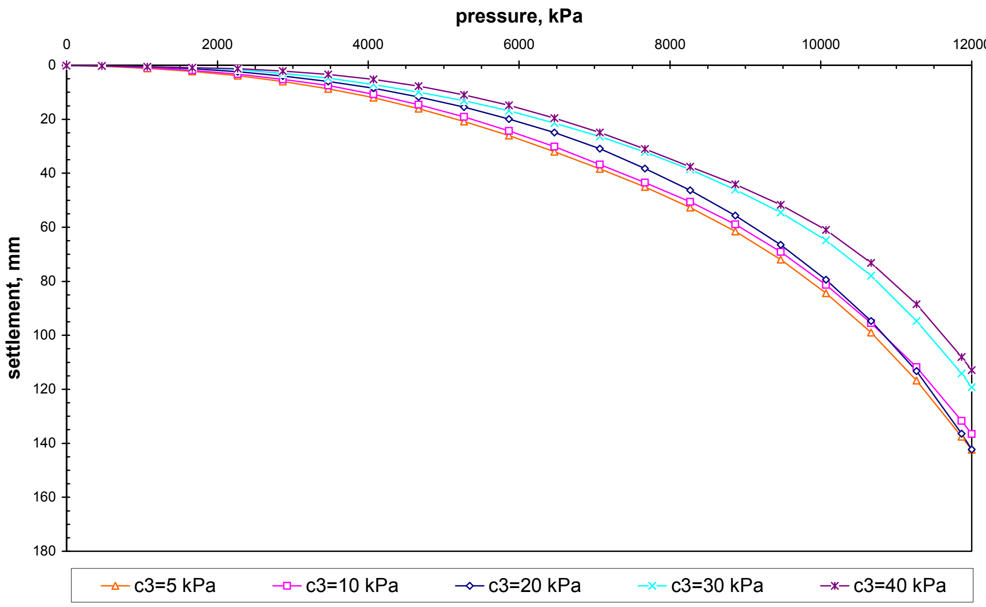

Fig. 18. Relation of pressure–settlement as a function of cohesion c_3 of interface layer (for $t = 0.0010$ m)

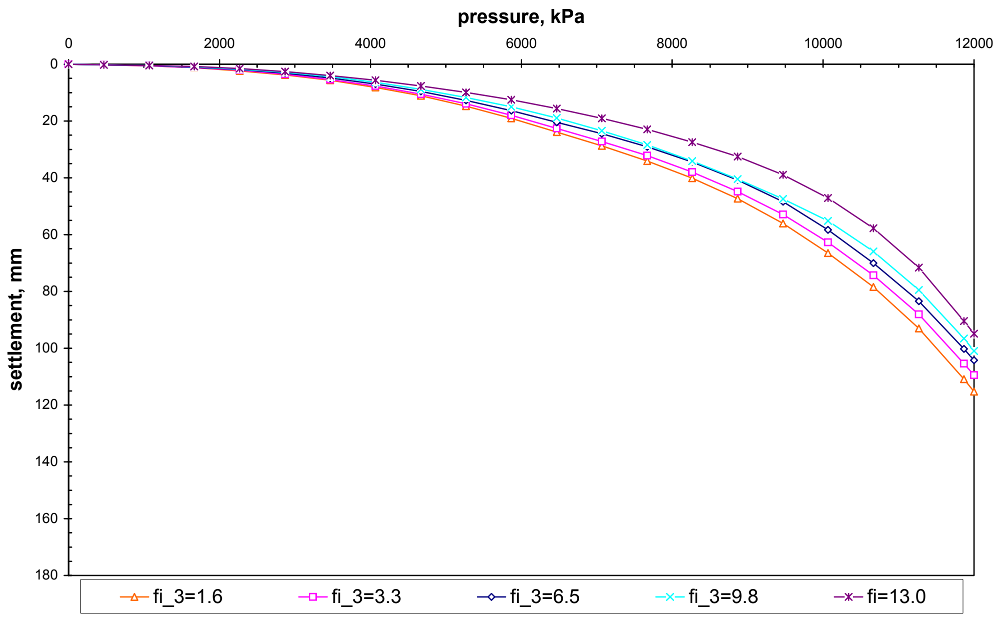

Fig. 19. Relation of pressure–settlement as a function of internal friction angle ϕ_3 of interface layer (for $t = 0.0005$ m)

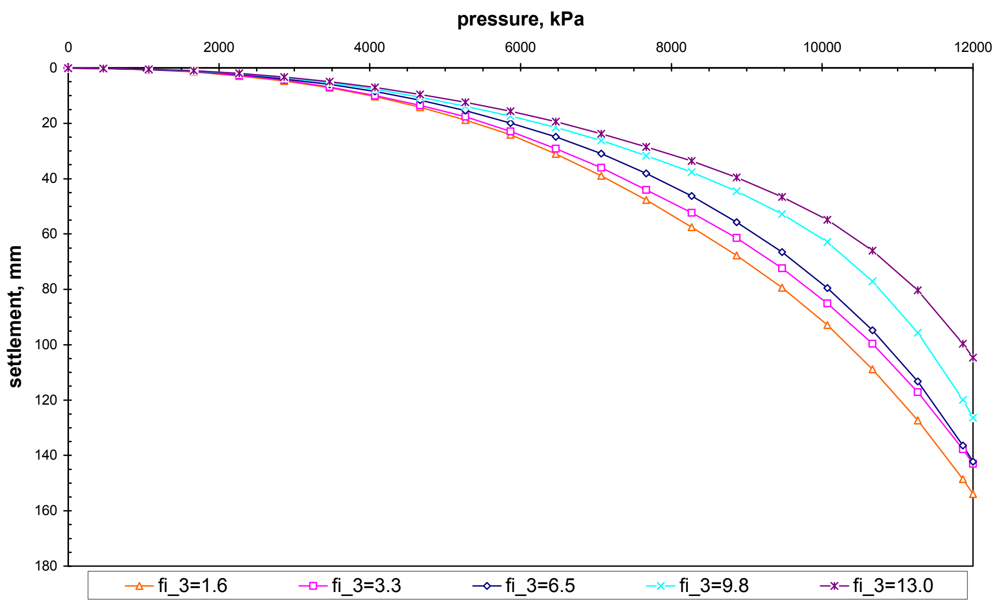

Fig. 20. Relation of pressure–settlement as a function of internal friction angle ϕ_3 of interface layer (for t = 0.0010 m)

Figure 21 shows the dependence of pile characteristics of pressure–settlement on contact zone thickness. The dependence was tested for eleven values, four of them were smaller and the other six bigger than the value assumed for jet-grouting pile.

So it is clear that an increase in thickness of contact layer is accompanied by an increase in pile settlement. Such an influence is enhanced as the load increases. Overlooking rather small perturbations resulting from numerical errors, the dependence can be regarded as linear (figure 22).

Quite interesting variant of sensitivity analysis is represented by the studies of the influence of consecutive changes of the contact layer material parameters on the settlement.

The results of such theoretical studies are shown in figures 23 and 24 in the form of pencil, depending on interpreted invariable the values of which give information about what part of E_2, c_2, ϕ_2, measured in proportion, is formed by material constants E_3, c_3, ϕ_3 of contact layer. There were analyzed characteristics for five values of integrated invariable within the range from 25% to 100% (to remind: the value of 67% corresponds with the characteristics $\overline{q} - s$ of tested jet-grouting pile).

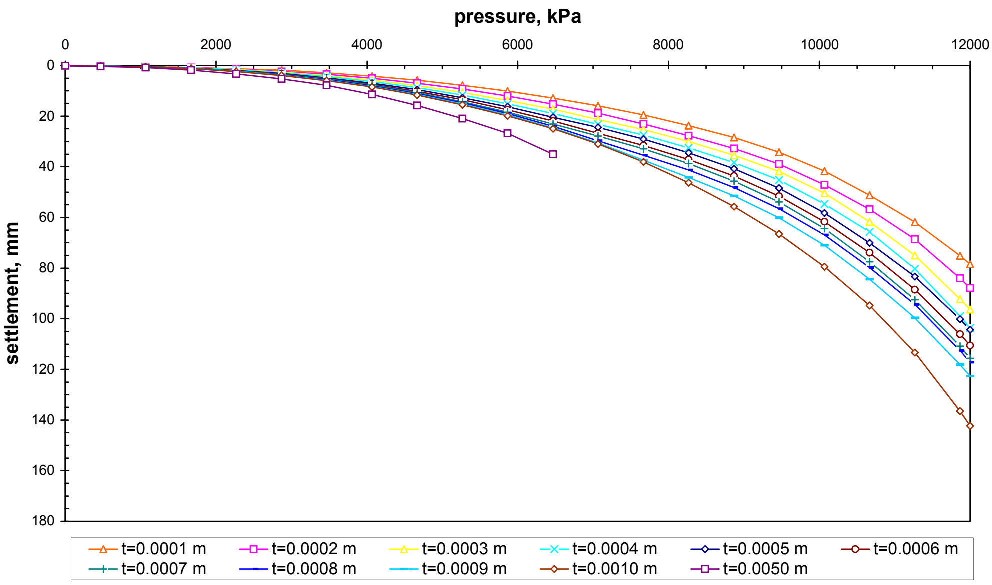

Fig. 21. Relation of pressure–settlement as a function of changes of thickness t of interface layer

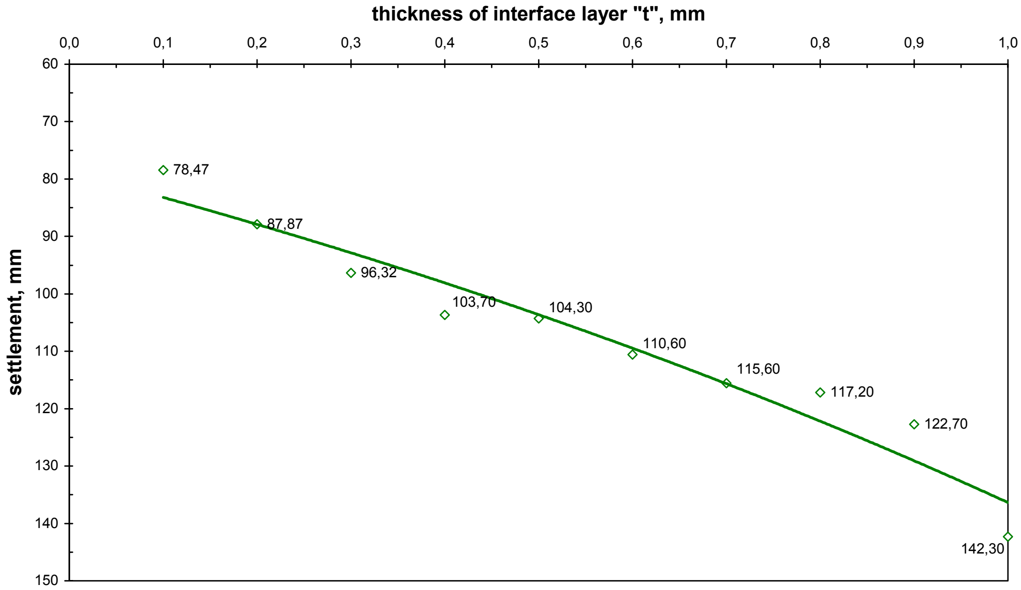

Fig. 22. Relation of thickness of interface layer *t*–settlement

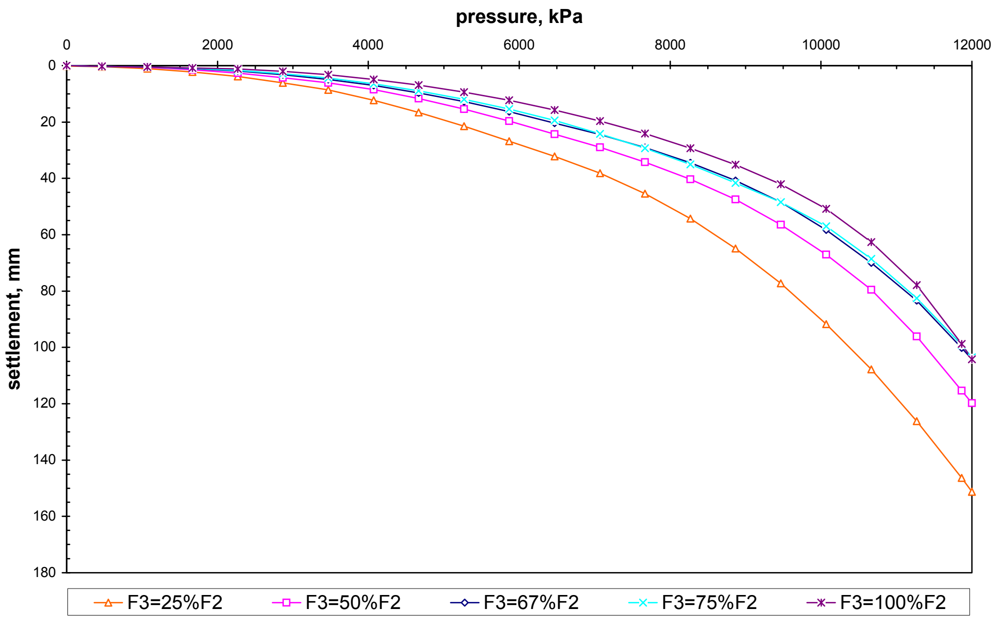

Fig. 23. Relation of pressure–settlement as a function of changes of interface layer's parameters (for t = 0.0005 m)

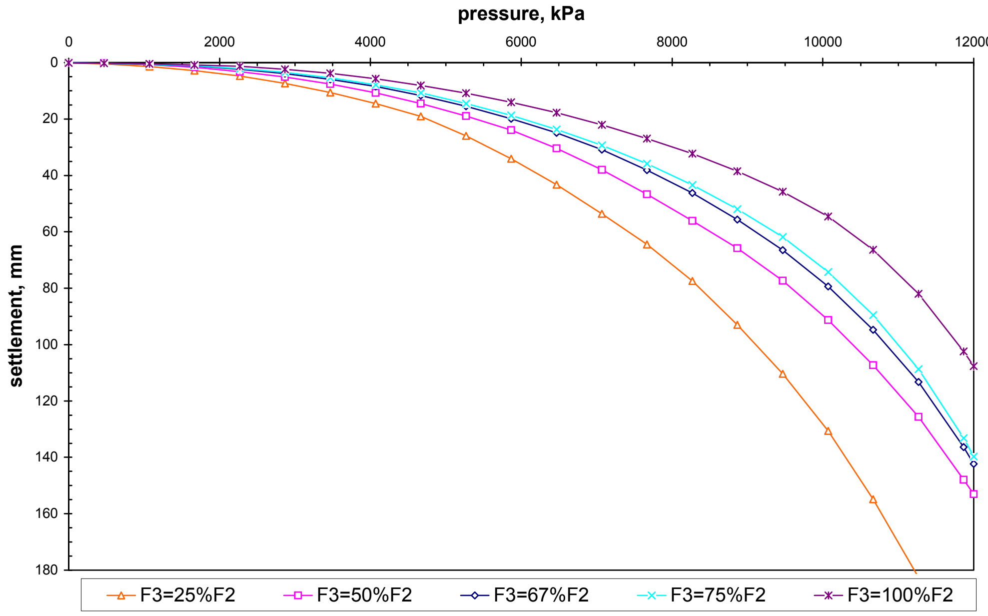

Fig. 24. Relation of pressure–settlement as a function of changes of interface layer's parameters (for $t = 0.0010$ m)

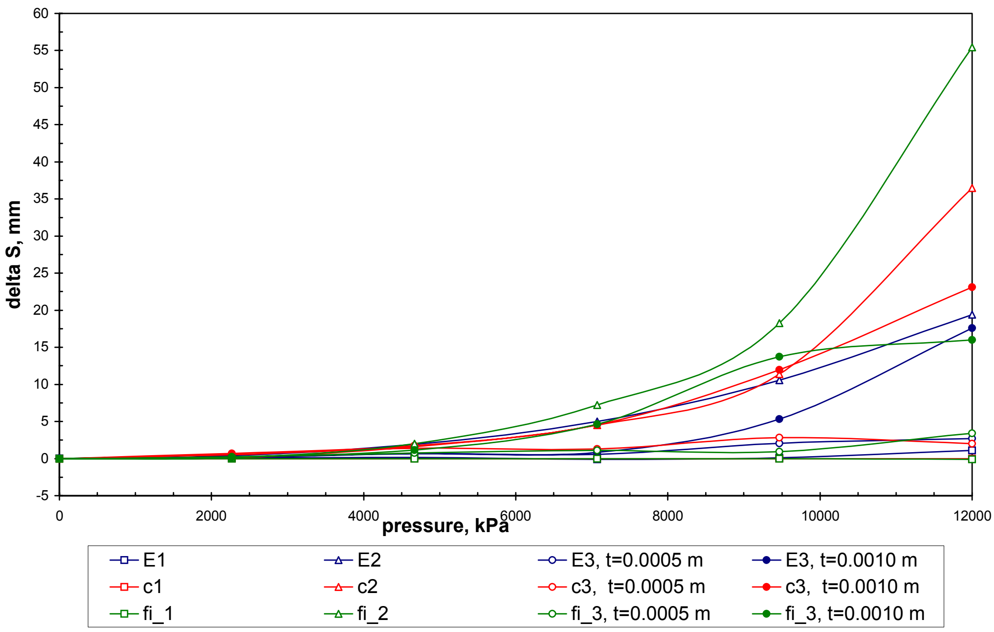

Fig. 25. Sensitivity of the relation of pressure–settlement of the jet-grouting pile model

to an increase in the values of parameters E_j, c_j, ϕ_j (j = 1, 2, 3) by 50%

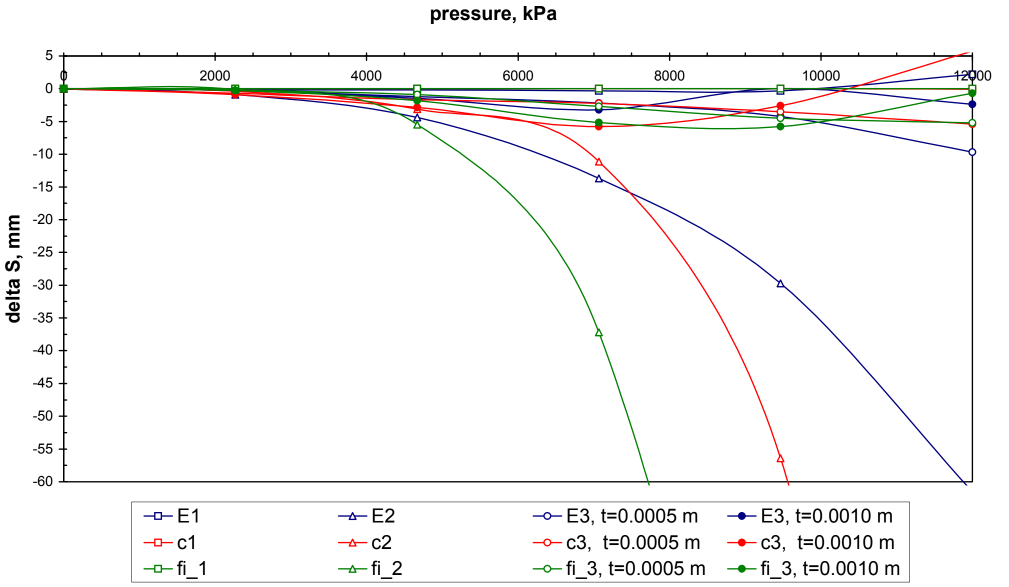

Fig. 26. Sensitivity of the relation of pressure–settlement of the jet-grouting pile model to a decrease in the values of parameters E_j, c_j, ϕ_j (j = 1, 2, 3) by 50%

General comments are similar to those in the case of separate changes of individual parameters. But unlike that case, settlement increases significantly at a relative decrease of contact parameters, especially in case of 1.0 mm zone, and at the same time the influence of a decrease is bigger than that of an increase of integrated variable in relation to its value for jet-grouting pile.

While stating more precisely a prior explanation of figures 25 and 26, the curves presented there are defined as graphic interpretation of deviation from the pile characteristics of pressure–settlement of parallel curves q–s, corresponding with an increase (figure 25) or a decrease (figure 26) in consecutive model parameters.

Comparing the influence of the changes of different model parameters on pile settlement in the same scale and in one picture allows changing either parameters of the same zone or one particular parameter for each of zones.

Careful analysis of figures 25 and 26 shows fundamental difference in sensitivity between different model zones. Sensitivity of subsoil is definitely bigger than that of contact layer, which, in turn, is by far bigger than sensitivity of pile material.

According to a general conclusion resulting from sensitivity analysis, there is a big difference in the pressure–settlement characteristics in the parameters of subsoil accepted for the analysis, especially in the case of internal friction angle and parameters of contact zone, the values of which are tightly related to subsoil parameters.

6. RECAPITULATION AND CONCLUSIONS

In the paper, a model of jet-grouting pile and interacting subsoil has been presented. The model was constructed in four stages: creation of the concept and its mathematical formulation, calibration, verification and analysis of its sensitivity.

The idea of the model is significantly different from quite few existing computational proposals which are adaptations of semi-empirical formulae used to calculate bearing capacity and settlement of classical reinforced concrete piles. The idea of bearing capacity is regarded in those proposals as limiting resistance of the soil surroundings to drilling in the pile. Maximum load, conditioned by the strength of reinforced concrete, is calculated independently, according to principles applied in designing pile structures, in complete isolation from the soil.

The concept presented in the paper represents phenomenological approach within mechanics of continuum or strictly speaking – theory of plasticity. Physical reality was modelled and concentric system of three interacted bodies of cylindrical shape and elastic-plastic properties was considered. The bodies represent: jet-grouting pile, subsoil adjacent to the base and side surface, as well as thin contact layer separated out of it. Geometric imperfections occurring inevitably in the contact zone as a derivative of execution technology were not taken into consideration.

To describe mechanical behaviour of each zone there was assumed the simplest constitutive model of incremental theory of plasticity, i.e. elastic-ideally plastic medium defined with the aid of the Coulomb–Mohr boundary state condition and associated with it flow law. The choice of a simple model was justified by pioneering character of the proposal and by practical need of using not many parameters with generally known physical meaning. The model was defined at the beginning as numerical and this involved adjusting its final form to static analyses by the method of finite elements. This meant that division of the model area into finite elements complied with the postulate of compatibility of zones' boundaries with elements boundaries and with other requirements of the method.

Almost independently of the level of complexity of the computational model for geotechnical structure, calibration of the model is a difficult task either technically or theoretically. It is also expensive and time-consuming. That was also the case of the proposal presented in this paper. Estimation of material and geometric parameters became a crucial research task in model creation. It was also vast (chapter 4) and complex in the aspect of methodology.

The methodology was conformed to two general ideas:

1. Coherent, own experimental database in the processes of calibration and verification of the model.

2. Separate estimation of parameters for each zone.

Coherent database was obtained by concentrating geometric and material experiments on one jet-grouting pile which was selected among some other tested on the field test of GEOREM in the city of Sosnowiec and on the soil from its close neighbourhood. The jet-grouting pile had been already exposed to trial load and the results were to be the basis of experimental model verification.

Verification plays a special role in the structure of every computational model as it provides the answer to a question whether and to what extent the model is effective in the aspect of possibilities of its simulation and use in practice. From the point of view based on this paper, this stage was aimed to prove the truth of the submitted proposition. It is worth reminding that the essence of the proposition in the paper was stating the model ability to predict realistically the pile settlement and its material effort in a wide range of load. The proof has a form of comparison of theoretical and experimental load–settlement characteristics of the tested jet-grouting pile shown in figures 7 and 7a. A theoretical characteristics is a result of increment-iterative MES analysis of the problem of pile interaction with the soil surroundings. In the analysis, the presence of contact layer and optimum calibration of the system numerical model, described in chapter 4, were taken into account. The experimental characteristics compares the results of measurements carried out during the trial load of the jet-grouting pile.

The comparison, as far as geotechnical structure is concerned, shows that the results of the calculations are in a strict conformity with measurements in a range of load up to 3700–3800 kPa. Above this level, increasing discrepancy between the characteristics appears. It can be said that the truth of the cautiously formulated idea of settlement has been hereby proved in this paper. Inability to extrapolate reliably the characteristics of trial load, which was not led up to the state of subsoil boundary bearing load due to technical reasons, does not allow us to evaluate the discrepancy between real pile bearing capacity and its prediction, with the use of three-zone, elastic-ideally plastic model described by flow law associated with the Coulomb–Mohr condition. The discrepancy is by no means significant and so, in this aspect, the model created in the paper is not more effective than the empirical formula by GWIZDAŁA and MOTAK [18].

Verification of the three-zone model of the jet-grouting pile–soil system, from the point of view of the ability to predict realistically stress pattern in the system and jet-grouting pile material efforts, would require to carry out independent experiments with the use of extensometry, radiometry etc., and/or FEM analysis of the system with the application of much more sophisticated elastic-plastic models with the parameters identified as precisely as possible, especially for the soil. Now they are hardly real tasks. It should be pointed out that the three-zone model presented is the first proposal which enables us to analyse stress pattern.

Finally, it is necessary to focus on the results of the analysis of the model sensitivity. It concerned only one but extremely important aspect, i.e. predicting the settlements of jet-grouting pile. General conclusion based on the numerical experiments conducted is that settlement is practically insensitive to the parameters of the material model of pile. However, the parameters of the model of soil massif and contact layer have significant influence on the size of settlement being a function of load, but not as big, so that it would question practical usefulness of the model. The above conclusions should be used in attempts to put the solution suggested into practice.

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