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# DETERMINATION OF MATERIAL COEFFICIENTS FOR THE THERMOMECHANICAL MODEL OF DRYING

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**Streszczenie:** Wilgotne materiały kapilarno-porowate zmieniają swe właściwości fizyczne podczas procesu suszenia. Pracę poświęcono badaniu zmian współczynników materiałowych występujących w termomechanicznym modelu suszenia w funkcji stanu zawilżenia. Zanalizowano współczynniki lepkosprężystego modelu Maxwella, w którym występują dwa współczynniki sprężystości, tj. moduł Younga $E$ i moduł ścinania $G$, oraz odpowiadające im dwa współczynniki lepkosprężystości $\Xi$ i $\eta$. Badania przeprowadzono na glince kaolinowej, korzystając z uniwersalnej maszyny wytrzymałościowej.

**Abstract:** Wet capillary-porous materials change their physical properties during drying. The aim of this paper was to examine the material coefficients in the thermomechanical model of drying as a function of moisture content. The coefficients appearing in the viscoelastic Maxwell model are analyzed. They are as follows: Young's modulus $E$, shear modulus $G$, and equivalent to them viscoelastic coefficients $\Xi$ and $\eta$. The studies were carried out on the kaolin clay using universal strength-measuring instrument.

**Резюме:** Влажные капилярно-пористые материалы изменяют свои физические свойства во время процесса сушки. Настоящая работа посвящена исследованиям материальных коэффициентов, выступающих в термомеханической модели сушки в функции состояния увлажнения. Проведен анализ коэффициентов вязкоупругой модели Максвелла, в которой выступают два коэффициента упругости, т.е. модуль Юнга $E$ и модуль сдвига $G$, а также два отечающих им коэффициента вязкоупругости $\Xi$ и $\eta$. Исследования проводились на каолине при помощи универсальной установки для прочностных испытаний.

## NOTATIONS

| | | |
|---|---|---|
| $s_{ij}$ | – deviatoric stress tensor | $N/m^2$, |
| $\sigma$ | – spherical stress tensor | $N/m^2$, |
| $e_{ij}$ | – deviatoric strain | 1, |
| $\varepsilon$ | – spherical strain | 1, |
| $\alpha_\vartheta$ | – thermal expansion coefficient | 1/deg, |
| $\alpha_\Theta$ | – moisture expansion (shrinkage) coefficient | 1, |
| $G$ | – shear modulus of elastic deformations | $N/m^2$, |
| $E$ | – Young's modulus of elastic deformations | $N/m^2$, |
| $\eta$ | – shear modulus of viscoelastic deformations | $N\,s/m^2$, |
| $\Xi$ | – viscoelastic modulus equivalent to elastic Young's modulus | $N\,s/m^2$, |
| $X$ | – moisture content | 1. |

## 1. INTRODUCTION

Wet capillary-porous materials change their physical properties during drying. At the beginning of a drying process, when the material is wet, viscoelastic and plastic physical relations better describe the properties and the behaviour of dried body than elastic relations. The first stage of drying is crucial for the quality of final products. Unfortunately, the use of more complex and sophisticated physical relations generates difficulties in modelling of drying problems. Moreover, there are no reliable material coefficients in recent scientific worldwide literature. For the elastic model these coefficients can be found for different materials; however, for other physical models they have to be determined on one's own in the laboratory.

The method presented in this article enables us to evaluate all elastic and viscoelastic coefficients used in Maxwell physical relation, with the help of universal strength-measuring instrument. The measuring instrument has to allow programming the experimental tests and carrying out the compression or shearing at different velocities. The studies were conducted on the kaolin clay with different moisture content.

## 2. PHYSICAL RELATION OF MAXWELL MODEL

Relations between stresses, deformations, temperature and moisture content in a dried body according to Maxwell model can be expressed in general form as follows

$$\dot{s}_{ij} + \frac{G}{\eta} s_{ij} = 2G\dot{e}_{ij},$$
$$\dot{\sigma} + \frac{K}{\kappa}\sigma = K(\dot{\varepsilon} - \dot{\varepsilon}^{(TX)}), \tag{1}$$

where $s_{ij}$ and $\sigma$ are the deviatoric and spherical parts of the total stress tensor; $e_{ij}$ is the strain deviator and $\varepsilon/3$ is the spherical part of the total strain; $K = GE/3(3G-E)$ is the bulk elastic modulus and $\kappa = \eta\Xi/3(3\eta - \Xi)$ is the viscous bulk modulus; $G$ and $E$ are the shear and Young's moduli; $\eta$ and $\Xi$ are the viscoelastic coefficients equivalent to shear and Young's moduli. In equation (1); $\varepsilon^{(TX)}$ can be termed as the volumetric deformation caused by the temperature and the moisture content:

$$\varepsilon^{(TX)} = 3(\alpha_{\vartheta}\vartheta + \alpha_{\Theta}\Theta), \tag{2}$$

where $\vartheta = T - T_0$ and $\Theta = X - X_0$ are the relative temperature and the relative moisture content, respectively.

Some researchers (ITAYA at al. [2], RAO [5]) use a simplified form of general Maxwell model in the drying theory. Namely, they neglect the influence of the viscous volumetric changes following the first Reiner's postulate. According to their suggestion the viscous strains appear only by deviatoric action of stresses. In our opinion, such a suggestion is true for non-porous bodies, but not for capillary-porous ones, where viscous effects may also affect volumetric changes.

## 3. MATERIAL

The studies were carried out on the kaolin clay. Two most common applications of kaolin are coating of paper to hide the pulp strands and production of high-grade ceramic products. Kaolin's whiteness, opacity and large surface area make it an ideal raw material for paper production. In this industry, kaolin is used both as a filler in the bulk of the paper and to coat its surface. Kaolin is widely used in manufacturing tableware, sanitaryware, and wall and floor tiles. It provides the strength and plasticity in the shaping of these products and reduces the amount of pyroplastic deformation in the process of firing. Besides of these two main applications, kaolin is also used in other industry branches. It can improve the optical, mechanical and rheological properties of a paint. Kaolin adds strength, abrasion resistance and rigidity to rubber. A major application of kaolin is in PVC cables where its main function is to improve the electrical resistivity. Due to its high rate of fire-proofness, kaolin is used to build structures subjected to high temperatures, ranging from simple to sophisticated products, e.g. from fireplace brick linings to the heat shields for the space shuttles. Kaolin is also known in cosmetics and pharmaceuticals industry. Composition of tested kaolin is presented in the table.

Table

KOC kaolin from Surmin-Kaolin S.A. company

| KOC Kaolin | | |
|---|---|---|
| Chemical constitution | $SiO_2$ | 51.3% |
| | $Al_2O_3$ | 34.3% |
| | $K_2O$ | 0.62% |
| | $Fe_2O_3$ | 0.50% |
| | $TiO_2$ | 0.49% |
| | MgO | 0.14% |
| | CaO | 0.07% |
| | $Na_2O$ | 0.01% |
| | ignition loss | 12.1% |
| Mineralogical composition | kaolinite | 80% |
| | illite | 9% |
| | quartz | 9% |
| | others | 2% |

## 4. DETERMINATION OF ELASTIC MATERIAL COEFFICIENTS

Shear modulus $G(X)$ and Young's modulus $E(X)$ for kaolin as a function of moisture content $X$ were determined experimentally by means of universal material strength-measuring instrument Koegel FGP 7/18 – 1000 (figure 1).

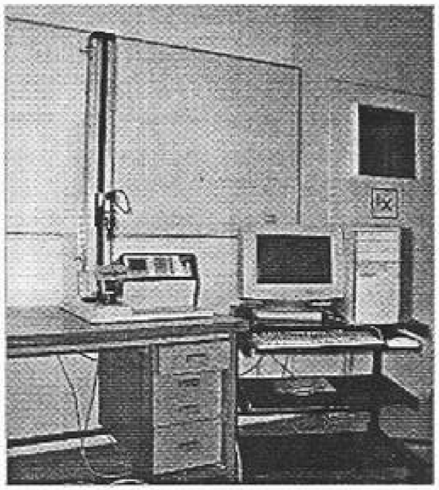

Fig. 1. Universal strength-measuring instrument Koegel FGP 7/18 – 1000

The values of Young's modulus were estimated in compressive tests for kaolin bar samples ($b \times h \times l$: 10 × 16 × 20 mm) whose moisture content ranged from 0 to 40%. The increment of compressive force in time was constant and equal to $\dot{P} = 1$ [N/s] during the whole test. The values of Young's modulus were estimated from the tangent of the angle between the horizontal axis and the tangent line to the early stage of the stress–strain curve. The results are presented in figure 2 (see also AUGIER et al. [1]).

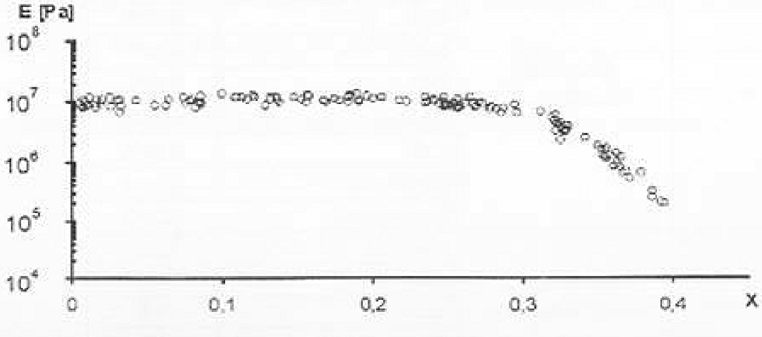


Fig. 2. Young's modulus for kaolin clay

It is seen from figure 2 that Young's modulus depends on moisture content, particularly for the moisture content greater than 30%. The kaolin becomes liquefied at the moisture content of ca. 50%.

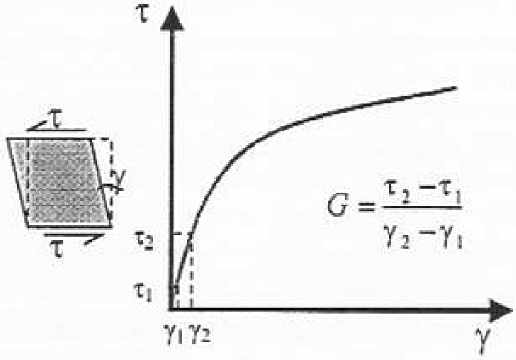


Fig. 3. Estimation of shear modulus

Shear modulus was determined in shear tests. The kaolin samples for shear tests were bars with rectangular cross-sections of 10×16 mm ($b\times h$) and 80 mm length. The gap between clamps was 6 mm, and the assumed loading force rate was equal to $\dot{P}=1$ [N/s]. The shear modulus was determined from the tangent of the angle between the $\gamma$-axis and the tangential line to the early stage of the stress–angle strain angle curve (figure 3). Additionally, the influence of non-uniform shear stress distribution in the rectangular cross-section and the influence of bending force on the gap length were considered. So, the evaluation of the shear modulus requires an exact knowledge of Young's modulus for a given moisture content. The value of final shear modulus was estimated based on equation (3)

$$G(X)=\frac{\beta P}{S\left(\gamma-\dfrac{Pl^2}{3E(X)I}\right)}, \tag{3}$$

where $\beta = 6/5$ is the coefficient of non-uniform shear stress distribution in the rectangular cross-section (KISIEL [3], ZIELNICA [6]), $P$ – applied force, $\gamma = f/l$ – shear strain, $f$ – deflection of the bar, $l$ – gap length beetwen clamps, $S = bh$ – cross-section area, $I = bh^3/12$ – section moment of interia.

Shear tests at different moisture contents have led to determination of the shear modulus presented in figure 4.

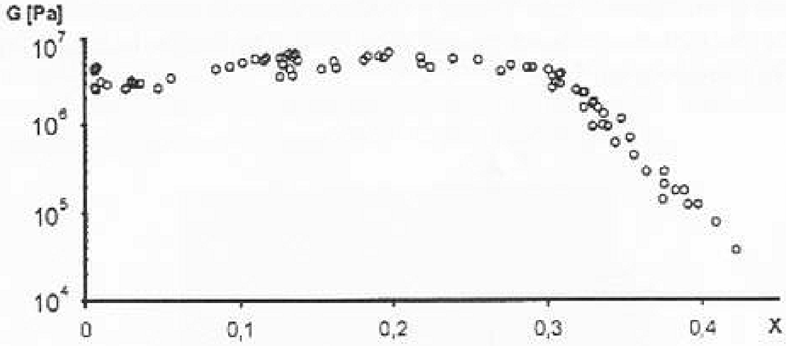


Fig. 4. Shear modulus for kaolin clay

Knowing Young's and shear moduli one can determine other commonly used elastic coefficients: Poisson ratio (4) and bulk modulus (5)

$$\nu(X) = \frac{E(X)}{2G(X)} - 1, \tag{4}$$

$$K(X) = \frac{G(X)E(X)}{3(3G(X) - E(X))}. \tag{5}$$

The results obtained in this way are presented in figure 5.

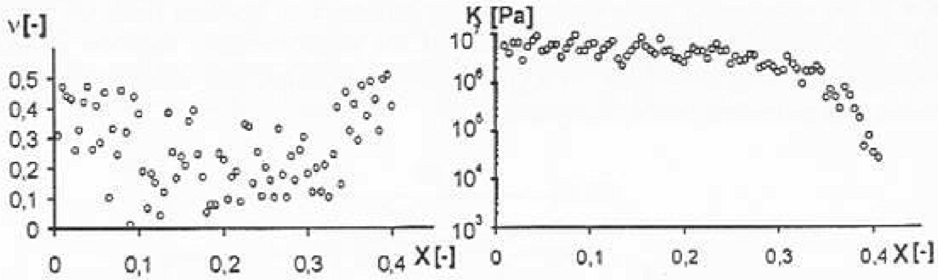


Fig. 5. Poisson's ratio $\nu(X)$ and the elastic bulk modulus $K(X)$ for kaolin clay

A big scattering of data, particulary for Poisson's ratio, follows from the fact that the errors of two experimental methods overlap in equations (4) and (5). To obtain more accurate values of Poisson's ratio and the elastic bulk modulus it is necessary to perform much more experiments.

## 5. DETERMINATION OF THE VISCOELASTIC COEFFICIENTS

An increase of moisture content in kaolin clay changes its physical properties. The material becomes more viscoelastic and its reological properties begin to dominate the other properties. In order to model these changes, the physical models based on linear viscoelastic theory (e.g. Maxwell, Kelvin–Voigt, standard, Birgham) are commonly used. In mechanics of non-porous media, Reiner's postulate is widelly applied. According to this postulate one assumes the occurrence of the viscoelastic strains in the deviatoric state of stress only. This simplifies the viscoelastic models since only one viscoelastic coefficient $\eta(X)$ is present. Our studies, however, deal with porous bodies, in which the volumetric creep is also possible. In such a case, there are two viscoelastic coefficients present in physical relation, namely: $\eta(X)$ and $\Xi(X)$, which are equivalent to shear modulus $G(X)$ and Young's modulus $E(X)$, respectively. The viscoelastic material coefficients are estimated based on the analysis of the experimental creep curves. Taking into account the experimental evolution of strain curves, one can find the viscoelastic coefficients for Maxwell model from the compressive and shear tests. For one-dimensional compressive test the Maxwel physical relation can be written as:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E(X)} + \frac{\sigma}{\Xi(X)}. \tag{6}$$

In our experimental method, the assumed stress increment in time is constant ($\dot{\sigma}$ = const), that is

$$\dot{\sigma} = c \tag{7a}$$

or

$$\sigma = ct + \sigma_0. \tag{7b}$$

The parabolic like function is a solution of equation (6) with the assumption (7b), namely

$$\varepsilon = \varepsilon_0 + c\left(\frac{1}{E(X)}t + \frac{1}{2\Xi(X)}t^2\right). \tag{8}$$

Interpolation of experimental data with the above approximation curve is presented in figure 6.

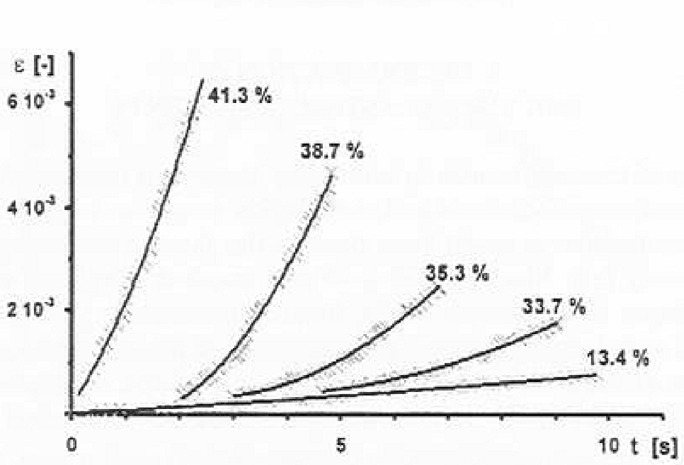


Fig. 6. Experimental (cross points) and theoretical (solid lines) strain curves evolution for different moisture content

Knowing Young's modulus for a particular moisture content one can calculate its viscoelastic equivalent by the best fitting of the theoretical curves to experimental ones (KOWALSKI [4]). Optimization of $\Xi(X)$ modulus values was done using the method of least squares. Figure 7 shows the final result.

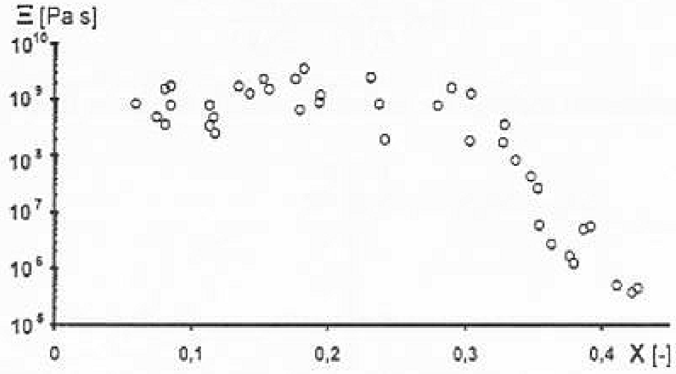


Fig. 7. Viscoelastic modulus $\Xi(X)$ for kaolin clay vs. moisture content

Analogical analysis can be performed for deviatoric stress state. The Maxwell model for simple shear test can be written as:

$$\dot{\gamma} = \frac{\dot{\tau}}{G(X)} + \frac{\tau}{\eta(X)}. \tag{9}$$

The determined viscoelastic coefficient $\eta(X)$ is shown in figure 8.

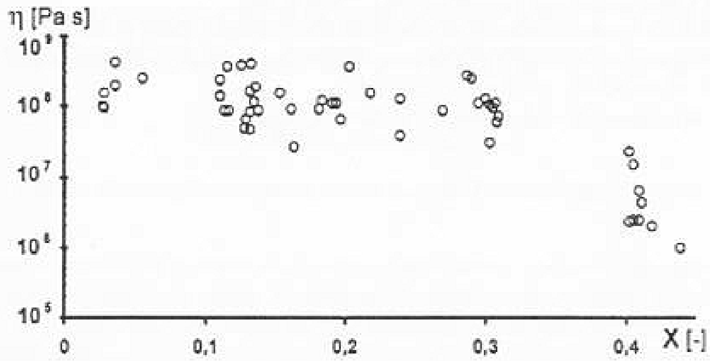


Fig. 8. Viscoelastic modulus $\eta(X)$ for kaolin clay vs. moisture content

One can calculate viscous changes for volumetric strains based on elastic-viscoelastic analogy principle:

$$\kappa(X) = \frac{\eta(X)(\Xi(X)}{3(3\eta(X) - \Xi(X))}.$$

## 6. CONCLUSIONS

Two kinds of experiments are required for determination of the material coefficients appearing in the Maxwell model for capillary-porous materials of different moisture contents. The elastic coefficients of kaolin samples were determined directly from the shear and the compressive tests for different moisture contents. It should be stressed that in these experiments there was assumed a constant force increment in time. This enabled estimation of viscous coefficients by matching up the curves representing the experimental strain evolutions with optimal theoretical ones using the method of least squares.

## ACKNOWLEDGEMENTS

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